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Contents

Key Words.
augmented Lagrangian method, nonsmooth optimization, Poisson denoising

AMS Subject Classifications.
49M37, 90C30, 90C48, 90C90

Abstract.
This paper shows that, for the stationary thermally coupled incompressible magnetohydrodynamics problem, an application of the grad-div stabilization technique and a modification of the Arrow–Hurwicz iteration can improve the convergence rate of the Arrow–Hurwicz algorithm and remove restrictions on the relaxation parameter α for this algorithm. Based on the grad-div stabilization method, we design an Arrow–Hurwicz-type iterative finite element algorithm for solving this problem. A convergence analysis as well as numerical tests show that the proposed iteration performs better for the considered problem compared to the standard Arrow–Hurwicz iteration.

Key Words.
Arrow–Hurwicz-type iteration, thermally coupled magnetohydrodynamics, grad-div stabilization, convergence analysis

AMS Subject Classifications.
65N12, 65N30

Abstract.
This paper provides a proof of global and quadratic convergence of the block Jacobi method for Hermitian matrices under the de Rijk pivot strategy. Also global and quadratic convergence of the element-wise Jacobi method under the same pivot strategy is proved. It is shown that sharp quadratic convergence bounds can be deduced from an estimate that is obtained in the global convergence proof. Numerical tests illustrate the behavior of the methods under the de Rijk pivot strategy.

Key Words.
eigenvalue problem, block Jacobi method, de Rijk pivot strategy, global convergence, quadratic convergence

AMS Subject Classifications.
65F15

Abstract.
An approach is proposed for solving large linear systems that combines Krylov methods with the use of two different grid levels. Eigenvectors are computed on the coarse grid and then used to deflate the eigenvalues on the fine grid with an

efficient projection. GMRES-type methods are first used on both the coarse and fine grids. Then another approach is given that uses a novel restarted BiCGStab (or IDR) method on the fine grid. While BiCGStab is generally considered to be a non-restarted method, restarting works well in this context. Tests show that this new approach can be very efficient for difficult linear equations problems.

Key Words.
linear equations, deflation, GMRES, BiCGStab, eigenvalues, two-grid

AMS Subject Classifications.
65F10, 15A06

Abstract.
We present a unified numerical method to determine the shapes of multiple Hele-Shaw bubbles in steady motion, and in the absence of surface tension, in three planar domains: free space, the upper half-plane, and an infinite channel. Our approach is based on solving the free boundary problem for the bubble boundaries using a fast and accurate boundary integral method. The main advantage of our method is that it allows for the treatment of a very high number of bubbles. The presented method is validated by recovering some existing results for steady bubbles in channels and free space. Several numerical examples are presented, many of which feature configurations of bubbles that have not appeared in the literature before.

Key Words.
Hele-Shaw flow, bubbles, free boundary problem, multiply connected domain, conformal mapping

AMS Subject Classifications.
76D27, 65E10

Abstract.
Recently, a new class of nonlinearly partitioned Runge–Kutta (NPRK) methods was proposed for nonlinearly partitioned systems of autonomous ordinary differential equations $y' = F(y, y)$. The target class of problems are those in which different scales, stiffnesses, or physics are coupled in a nonlinear way, wherein the desired partition cannot be written in a classical additive or component-wise fashion. Here we use a rooted-tree analysis to derive full-order conditions for NPRK_M methods, where M denotes the number of nonlinear partitions. Due to the nonlinear coupling and thereby the mixed product differentials, it turns out that the standard node-colored rooted-tree analysis used in analyzing ODE integrators does not naturally apply. Instead we develop a new edge-colored rooted-tree framework to address the nonlinear coupling. The resulting order conditions are enumerated, are provided directly for up to fourth order with $M = 2$ and third order with $M = 3$, and are related to existing order conditions of additive and partitioned RK methods. We conclude with an example that shows how the nonlinear order conditions can be used to obtain an embedded estimate of the state-dependent nonlinear coupling strength in a dynamical system.

Key Words.
Runge–Kutta, order conditions, time integration, nonlinear coupling

AMS Subject Classifications.
65L05, 65L06, 65L70

Abstract.
Recently Ahmadi et al. [IEEE Trans. Parallel Distrib. Syst., 32 (2021), pp. 1452–1464] and Tagliaferro [Research Square (2022)] proposed some iterative methods for the numerical solution of linear systems which, under the classical hypothesis of strict diagonal dominance, typically converge faster than the Jacobi method but slower than the forward/backward Gauss–Seidel one. In this paper we introduce a general class of iterative methods, based on suitable splittings of the matrix that defines the system, which include all of the methods mentioned above and have the same cost per iteration in a sequential computation environment. We also introduce a partial order relation in the set of splittings and, partly theoretically and partly on the basis of a number of examples, we show that such partial order is typically connected to the speed of convergence of the corresponding methods. We pay particular attention to the case of linear systems for which the Jacobi iteration matrix is nonnegative, in which case we give a rigorous proof of the correspondence between the partial order relation and the magnitude of the spectral radius of the iteration matrices. Within the considered general class, some new specific promising methods are proposed as well.

Key Words.
linear systems, iterative methods, matrix splitting

AMS Subject Classifications.
65F10

Abstract.
In this paper, we consider the tensor fixed-rank least-squares problem arising in image restoration. The Riemannian conjugate gradient method with an exact line search technique is designed to solve this problem. A convergence analysis of this method is given. Numerical experiments with synthetic data and real images demonstrate the feasibility and effectiveness of the new method.

Key Words.
tensor least-squares problem, fixed-rank constraint, Riemannian conjugate gradient method, exact line search, convergence analysis

AMS Subject Classifications.
15A69, 58C05, 65F10

Abstract.
We propose RTSMS (Randomized Tucker with Single-Mode-Sketching), a randomized algorithm for approximately computing a low-rank Tucker decomposition of a given tensor. It uses sketching and the least-squares method to compute the Tucker decomposition in a sequentially truncated manner. RTSMS essentially only sketches one mode at a time, so the sketch matrices are significantly smaller than for alternative approaches. It uses a rank estimator to adaptively find an appropriate rank for the Tucker decomposition, without requiring it as input. RTSMS is demonstrated to be competitive with existing methods, sometimes outperforming them by a large margin.

Key Words.
tensor decompositions, randomized algorithms, sketching, least-squares, leverage scores, Tikhonov regularization, iterative refinement, HOSVD

AMS Subject Classifications.
68W20, 65F55, 15A69

Abstract.
This paper analyzes direct solvers for centrosymmetric linear systems by applying structure-preserving factorizations with a particular focus on assessing their stability. We build on existing algorithms and complement the factorizations with equilibration and mixed-precision computations. The solvers are applied to linear systems arising from spectral discretizations of partial differential equations, and the results demonstrate their effectiveness. We evaluate the accumulation of roundoff errors during the computational process and their impact on the numerical solution. The study demonstrates that errors originating from the factorization of the matrix and a modified substitution propagate in a stable manner, establishing the direct solver's robustness. Additionally, we provide insights into the solver's stability by proving a bound for the relative error.

Key Words.
direct solution of linear systems, centrosymmetric and skew-centrosymmetric matrices, numerical stability, mixed precision

AMS Subject Classifications.
65F05, 65G50, 65N35

Abstract.
This paper describes software for computing a few of the largest or smallest singular triplets of a third-order tensor using the t-product. The software implements restarted partial tensor bidiagonalization techniques that were introduced by El Hachimi et al. in [Numer. Linear Algebra Appl., 31 (2024), Art. e2530]. Restarting is carried out by augmenting the available solution subspace by Ritz lateral slices or by harmonic

Ritz lateral slices. The performance of our Python implementation is investigated. The software is designed for easy use in various applications.

Key Words.
tensor algorithms, singular triplets, t-product, Ritz augmentation, harmonic Ritz augmentation

AMS Subject Classifications.
15A69, 65F15, 65F50

Abstract.
We study the stability of a two-dimensional Lax-Wendroff scheme in a quarter-plane. Following our previous work in [Commun. Math. Sci., 21 (2023), pp. 2051–2082], we aim here at adapting the energy method in order to study *second-order* extrapolation boundary conditions. We first show, based on the one-dimensional problem, why modifying the energy is a necessity in order to obtain stability estimates. We then study the two-dimensional case and propose a modified energy as well as second-order extrapolation boundary and corner conditions in order to maintain second-order accuracy and stability of the whole scheme, including near the corner.

Key Words.
transport equations, numerical schemes, domains with corners, boundary conditions, stability

AMS Subject Classifications.
65M12, 65M06, 65M20

Abstract.
Using a recently derived integral representation in terms of elementary functions, we give new asymptotic expansions of the normal inverse Gaussian cumulative distribution function. One of its asymptotic representations is stated in terms of the normal Gaussian distribution or complementary error function.

Key Words.
normal inverse Gaussian distribution, asymptotic analysis, error function

AMS Subject Classifications.
41A60, 33B20, 62E20, 65D20

Abstract.
This paper presents a model for tumor growth using a nonlocal velocity. We establish some results on the existence and uniqueness of solutions for a nonlocal tumor growth model. Many experiments show that the tumor spheroid can be invariant

under rotation and can maintain the shape of a spheroid during the growth process in some particular cases. Here, we assume that the multiple components of the system are invariant under rotation. Then, we use the collocation method to solve the nonlocal system. To illustrate the efficiency of the proposed method, we performed numerical tests that simulate a tumor growth scenario.

Key Words.
collocation method, nonlocal tumor growth model, partial differential equations

AMS Subject Classifications.
35R09, 65M70, 92C50, 35Q92

401 Structured backward errors of sparse generalized saddle point problems with Hermitian block matrices.
Sk. Safique Ahmad and Pinki Khatun.

Abstract.
In this paper, we derive the structured backward error (BE) for a class of generalized saddle point problems (GSPPs) with perturbations preserving the sparsity pattern and the Hermitian structures of the block matrices. Additionally, we construct the optimal backward perturbation matrices for which the structured BE is achieved. Our analysis also examines the structured BE in cases where the sparsity pattern is not maintained. Through numerical experiments, we demonstrate the reliability of the obtained structured BEs and the corresponding optimal backward perturbations. Finally, the computed structured BEs are used to assess the strong backward stability of some numerical methods used to solve the GSPP.

Key Words.
Hermitian matrices, backward error, perturbation analysis, saddle point problems, sparsity

AMS Subject Classifications.
15A12, 65F20, 65F35, 65F99

424 Revisiting the notion of approximating class of sequences for handling approximated PDEs on moving or unbounded domains.
Andrea Adriani, Alec Jacopo Almo Schiavoni-Piazza, Stefano Serra-Capizzano, and Cristina Tablino-Possio.

Abstract.
In the current work we consider matrix sequences $\{B_{n,t}\}_n$, with matrices of increasing sizes, depending on n, and equipped with a parameter $t > 0$. For every fixed $t > 0$, we assume that each $\{B_{n,t}\}_n$ possesses a canonical spectral/singular values symbol f_t, defined on $D_t \subset \mathbb{R}^d$, which are sets of finite measure, for $d \geq 1$. Furthermore, we assume that $\{\{B_{n,t}\}_n : t > 0\}$ is an approximating class of sequences (a.c.s.) for $\{A_n\}_n$ and that $\bigcup_{t>0} D_t = D$ with $D_{t+1} \supset D_t$. Under such assumptions and via the notion of a.c.s, we prove results on the canonical distributions of $\{A_n\}_n$, whose symbol, when it exists, can be defined on the, possibly unbounded, domain D of finite or even infinite measure.
We then extend the concept of a.c.s. to the case where the approximating sequence $\{B_{n,t}\}_n$ has possibly a different dimension than the one of $\{A_n\}_n$. This concept seems to be particularly natural when dealing, e.g., with the approximation both of

a partial differential equation (PDE) and of its (possibly unbounded or moving) domain D, using an exhausting sequence of domains $\{D_t\}$. Examples coming from approximated PDEs with either moving or unbounded domains are presented in connection with the classical and the new notion of a.c.s., while numerical tests and a list of open questions conclude the present work.

Key Words.
discretization of PDEs, moving/unbounded domains, spectral distribution of matrix sequences, (generalized) approximating class of sequences, generalized locally Toeplitz (GLT) matrix sequences, GLT theory

AMS Subject Classifications.
15A18, 65N, 47B

452 Asymptotic estimates of the error bound for Gauss–Radau quadratures.
Eleonora Denich.

Abstract.
This paper deals with the derivation of asymptotic expressions for the quadrature error of Gauss–Radau–Jacobi and Gauss–Radau–Laguerre formulas. Starting from the contour integral representation of the remainder term, the analysis is derivative-free and based on the theory of analytic functions. The final error estimates allow to select a-priori the number of quadrature points necessary to achieve a prescribed accuracy. Several numerical examples are reported.

Key Words.
Gauss–Radau rule, contour integral representation, error estimate

AMS Subject Classifications.
65D32, 33C45

468 An approach to discrete operator learning based on sparse high-dimensional approximation.
Daniel Potts and Fabian Taubert.

Abstract.
We present a dimension-incremental method for function approximation in bounded orthonormal product bases to learn the solutions of various differential equations. Therefore, we decompose the source function of the differential equation using parameters like Fourier or spline coefficients and treat the solution of the differential equation as a high-dimensional function with respect to the spatial variables, to these parameters, and also to further possible parameters from the differential equation itself. Finally, we learn this function in the sense of sparse approximation in a suitable function space by detecting coefficients of the basis expansion with the largest absolute values. Investigating the corresponding indices of the basis coefficients yields further insights into the structure of the solution as well as its dependency on the parameters and their interactions. This allows a reasonable generalization to even higher dimensions and therefore better resolutions of the decomposed source function.

Key Words.
sparse approximation, nonlinear approximation, high-dimensional approximation, dimension-incremental algorithm, partial differential equations, operator learning

Petr Vacek, Jan Papež, and Zdeněk Strakoš.

Abstract.

Multilevel methods represent a powerful approach to the numerical solution of partial differential equations. The multilevel structure can also be used to construct estimates for the total and algebraic errors of the computed approximations. This paper deals with residual-based error estimates that rely on properties of quasi-interpolation operators, stable splittings, or frames. We focus on the settings where the system matrix on the coarsest level is still large and the associated terms in the estimates can only be approximated. We show that the way in which the error term associated with the coarsest level is approximated is crucial. It can significantly affect both the efficiency (accuracy) of the overall error estimates and their robustness with respect to the size of the coarsest-level problem. We propose a new approximation of the coarsest-level term based on using the conjugate gradient method with an appropriate stopping criterion. We prove that the resulting estimates are efficient and robust with respect to the size of the coarsest-level problem. Numerical experiments illustrate the theoretical findings.

Key Words.

a posteriori estimates, multilevel hierarchy, residual-based error estimator, large coarsest-level problem, iterative computation

AMS Subject Classifications.

65N15, 65N55, 65N22, 65N30,65F10

609 On Runge–Kutta methods of order 10.
Misha Stepanov.

Abstract.

An explicit s-stage Runge–Kutta method of order 10 is determined by $s(s+1)/2$ parameters that must satisfy a non-linear algebraic system of 1205 equations. In the literature, solutions for the cases $s = 18$ [A. R. Curtis, J. Inst. Math. Appl., 16 (1975), pp. 35–55] and $s = 17$ [E. Hairer, J. Inst. Math. Appl., 21 (1978), pp. 47–59] were analytically derived, while that for $s = 16$ [D. K. Zhang, Numer. Algorithms, 96 (2024), pp. 1243–1267] was found by numerical search. In the present paper, a family of methods with $s = 15$ is derived.

Key Words.

minimal number of stages, explicit Runge–Kutta methods

AMS Subject Classifications.

65L05, 65L06