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Contents

Abstract.
This paper deals with the problem of numerically computing the roots of polynomials $p_k(x)$, $k = 1, 2, \ldots$, of degree $n = 2^k - 1$ recursively defined by $p_1(x) = x + 1$, $p_k(x) = xp_{k-1}(x)^2 + 1$. An algorithm based on the Ehrlich-Aberth simultaneous iterations complemented by the Fast Multi-pole Method (FMM) and the fast search of near neighbors of a set of complex numbers is provided. The algorithm, which relies on a specific strategy of selecting initial approximations, costs $O(n \log n)$ arithmetic operations per step. A Fortran 95 implementation is given and numerical experiments are carried out. Experimentally, it turns out that the number of iterations needed to arrive at numerical convergence is $O(\log n)$. This allows us to compute the roots of $p_k(x)$ up to degree $n = 2^{24} - 1$ in about 16 minutes on a laptop with 16 GB RAM, and up to degree $n = 2^{28} - 1$ in about 69 minutes on a machine with 256 GB RAM. The case of degree $n = 2^{30} - 1$ would require more memory and higher precision to separate the roots. With a suitable adaptation of the FMM to the limit of 256 GB RAM and by performing the computation in extended precision (i.e. with 10-byte floating point representation) we were able to compute all the roots in about two weeks of CPU time for $n = 2^{30} - 1$. From the experimental analysis, explicit asymptotic expressions of the real roots of $p_k(x)$ and an explicit expression of $\min_{i \neq j} |\xi_i^{(k)} - \xi_j^{(k)}|$ for the roots $\xi_i^{(k)}$ of $p_k(x)$ are deduced. The approach is effectively applied to general classes of polynomials defined by a doubling recurrence.

Key Words.
Mandelbrot polynomials, polynomial roots, Ehrlich-Aberth iteration, fast multipole method

AMS Subject Classifications.
65H04, 37F10, 30C15

Abstract.
In this paper, we propose a compound scheme of different product integration rules for the simultaneous approximation of both Hilbert and Hadamard transforms of a given function f. The advantages of such a scheme are multiple: a saving in the number of function evaluations and the avoidance of the derivatives of the density function f when approximating the Hadamard transform. Stability and conver-

gence of the proposed method are proved in the space of locally continuous functions in $(-1,1)$ with possible algebraic singularities at the endpoints, equipped with weighted uniform norms. The theoretical estimates are confirmed by several numerical tests.

Key Words.
hypersingular integrals, finite Hilbert transform, Hadamard finite part integrals, polynomial approximation, extended Lagrange interpolation, orthogonal polynomials

AMS Subject Classifications.
65D32, 65R10, 41A10, 41A28, 44A15

51 Averaged Nyström interpolants for bivariate Fredholm integral equations on the real positive semi-axes.
Dušan LJ. Djukić, Luisa Fermo, and Rada M. Mutavdžić Djukić.

Abstract.
Nyström interpolants based on suitable anti-Gauss cubature formulae associated with the Laguerre weights are provided for the numerical solution of second-kind Fredholm integral equations defined on the first quadrant in the coordinate plane $(0,\infty)\times(0,\infty)$. The case when the right-hand side and the kernel may increase at the origin and/or at infinity is considered. Numerical tests illustrate the good performance of such interpolants.

Key Words.
second-kind Fredholm integral equations, anti-Gauss cubature formulae, averaged cubature rules, Nyström methods

AMS Subject Classifications.
65R20, 65D32, 65D30

66 Software for limited memory restarted ℓ^p-ℓ^q minimization methods using generalized Krylov subspaces.
Alessandro Buccini and Lothar Reichel.

Abstract.
This paper describes software for the solution of finite-dimensional minimization problems with two terms, a fidelity term and a regularization term. The sum of the p-norm of the former and the q-norm of the latter is minimized, where $0 < p, q \leq 2$. We note that the "p-norm" is not a norm when $0 < p < 1$, and similarly for the "q-norm". This kind of minimization problems arises when solving linear discrete ill-posed problems, such as certain problems in image restoration. They also find applications in statistics. Recently, limited-memory restarted numerical methods that are well suited for the solution of large-scale minimization problems of this kind were described by the authors in [Adv. Comput. Math., 49 (2023), Art. 26]. These methods are based on the application of restarted generalized Krylov subspaces. This paper presents software for these solution methods.

Key Words.
ℓ^p-ℓ^q minimization, inverse problem, regression, iterative method

AMS Subject Classifications.
65F10, 65R32, 90C26

Abstract.
When Gauss quadrature rules are applied, the weight function is usually assumed to be nonnegative on the interval of integration. This paper considers recently introduced Gauss-type quadrature formulas with respect to weight functions that change sign in the interior of the interval of integration. To economically estimate the error of these formulas, we propose extensions based on Gauss-Kronrod, averaged Gauss, and generalized averaged Gauss quadrature rules. Numerical examples illustrate the accuracy of the introduced error estimates.

Key Words.
Gauss quadrature formula, variable-sign weight function, error estimate

AMS Subject Classifications.
65D30, 65D32, 41A55

Abstract.
In this paper we introduce a two-dimensional first-kind integral model to describe the interaction between the soil and an electromagnetic device. This model is used to reconstruct the electrical conductivity of the soil from electromagnetic data. The definition of the two-dimensional model is derived, and a numerical study of the forward model based on Gauss–Legendre quadrature formulae is presented. To solve the inverse problem, a linear system obtained from the discretization of the integral equation in the model is considered. The main difficulty is the severe ill-conditioning of the system, so the Tikhonov regularization method is applied and different regularization matrices and choice-rules for the regularization parameter are proposed. Several numerical tests show the effectiveness of the proposed approach.

Key Words.
first-kind integral equations, Gauss–Legendre quadrature, Tikhonov regularization, electromagnetic data

AMS Subject Classifications.
65R20, 65D32, 65Z05, 65F22

Abstract.
Many problems in scientific computing require the evaluation of Gauss quadrature rules. It is important to be able to estimate the quadrature error in these rules. Error estimates or error bounds often can be computed by evaluating an additional related Gauss-type formula such as a Gauss-Radau, Gauss-Lobatto, anti-Gauss, averaged Gauss, or optimal averaged Gauss rule. This paper presents software for both the evaluation of a single Gauss quadrature rule and the calculation of a pair of a Gauss rule and a related Gauss-type rule. The software is based on a divide-and-conquer method. This method is compared to both an available and a new implementation

of the Golub-Welsch algorithm, which is the classical approach to evaluate a single Gauss quadrature rule. Timings on a laptop computer show the divide-and-conquer method to be competitive except for the computation of a single quadrature rule with very few nodes.

Key Words.
quadrature, Gauss rule, Gauss-Radau rule, Gauss-Lobatto rule, averaged Gauss rule, optimal averaged Gauss rule quadrature, divide-and-conquer method, Golub-Welsch algorithm

AMS Subject Classifications.
65D30, 65D32

137 A stable numerical method for integral epidemic models with behavioral changes in contact patterns.
Bruno Buonomo, Eleonora Messina, Claudia Panico, and Antonia Vecchio.

Abstract.
We propose a non-standard numerical method for the solution of a system of integro-differential equations describing an epidemic of an infectious disease with behavioral changes in contact patterns. The method is constructed in order to preserve the key characteristics of the model, like the positivity of solutions, the existence of equilibria, and asymptotic behavior. We prove that the numerical solution converges to the exact solution as the step size h of the discretization tends to zero. Furthermore, the method is first-order accurate, meaning that the error in the discretization is $O(h)$, it is linearly implicit, and it preserves all the properties of the continuous problem, unconditionally with respect to h. Numerical simulations show all these properties and confirm, also by means of a case-study, that the method provides correct qualitative information at a low computational cost.

Key Words.
epidemic models, integro-differential equations, non-standard finite difference scheme, discrete models, perturbation theory

AMS Subject Classifications.
45D05, 65R20, 39A12, 92D30.

157 Internality of two-measure-based generalized Gauss quadrature rules for modified Chebyshev measures.
Dušan Lj. Djukić, Rada M. Mutavdžić Djukić, Aleksandar V. Pejčev, Lothar Reichel, Miodrag M. Spalević, and Stefan M. Spalević.

Abstract.
Many applications in science and engineering require the approximation of integrals of the form $\int_{-1}^{1} f(x)d\sigma(x)$, where f is an integrand and $d\sigma$ is a nonnegative measure. Such approximations often are computed by an ℓ-node Gauss quadrature rule $G_\ell(f)$ that is determined by the measure. It is important to be able to estimate the quadrature error in these approximations. Error estimates can be computed by applying another quadrature rule, $Q_m(f)$, with $m > \ell$ nodes, and using the difference $Q_m(f) - G_\ell(f)$ as an estimate for the error in $G_\ell(f)$. This paper considers the situation when $d\sigma$ is a modified Chebyshev measure and shows that two-measure-based quadrature rules $\widehat{Q}_{2\ell+1}$ exist, have positive weights, and have distinct nodes in the interval $[-1, 1]$. The last property makes them applicable also

when the integrand f only is defined in $[-1, 1]$. Comparisons with other choices of quadrature formulas $Q_{2\ell+1}$ are presented. This paper extends the investigation of two-measure-based quadrature rules for Jacobi and generalized Laguerre measures initiated in A. V. Pejčev et. al [Appl. Numer. Math., 204 (2024), pp. 206–221].

Key Words.
Gauss quadrature rule, averaged Gauss rule, generalized averaged Gauss rule, modified Chebyshev measure

AMS Subject Classifications.
65D30, 65D32, 33C45, 33C47

173 Linear FDEM subsoil data inversion in Banach spaces.
P. Díaz de Alba, C. Estatico, M. Lazzaretti, and G. Rodriguez.

Abstract.
The applicative motivation of this paper is the reconstruction of some electromagnetic features of the earth superficial layer by measurements taken above the ground. We resort to frequency domain electromagnetic data inversion through a well-known linear integral model by considering three different collocation methods to approximate the solution of the continuous problem as a linear combination of linearly independent functions. The discretization leads to a strongly ill-conditioned linear system. To overcome this difficulty, an iterative regularization method based on Landweber iterations in Banach spaces is applied to reconstruct solutions which present discontinuities or have a low degree of smoothness. This kind of solutions are common in many imaging applications. Several numerical experiments show the good performance of the algorithm in comparison to other regularization techniques.

Key Words.
first-kind Fredholm integral equations, electromagnetic induction, inverse problems in geophysics, collocation methods, iterative regularization, Landweber method, Banach spaces

AMS Subject Classifications.
45B05, 65F22, 65R20, 86A22