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speed-ups in applications involving parabolic equations, its effectiveness in the hyperbolic framework remains debatable: growth of instabilities and slow convergence are both strong issues in this case, which often negate most of the advantages from time-parallelisation. Here, we focus on the Multigrid Reduction in Time algorithm, investigating in detail its performance when applied to non-linear conservation laws with a variety of discretisation schemes. Specific attention is given to high-accuracy Weighted Essentially Non-Oscillatory reconstructions, coupled with Strong Stability Preserving integrators, which are often the discretisations of choice for such equations. A technique to improve the performance of MGRIT when applied to a low-order, more dissipative scheme is also outlined. This study aims at identifying the main causes for degradation in the convergence speed of the algorithm and finds the Courant-Friedrichs-Lewy limit to be the principal determining factor.

Key Words.
parallel-in-time integration, multigrid, conservation laws, WENO, high-order methods.

AMS Subject Classifications.
65M08, 35L65, 65M55, 65Y05, 65Y20

Abstract.
The balancing domain decomposition by constraints (BDDC) method is applied to the linear system arising from the finite volume element method (FVEM) discretization of a scalar elliptic equation. The FVEMs share nice features of both finite element and finite volume methods and are flexible for complicated geometries with good conservation properties. However, the resulting linear system usually is asymmetric. The generalized minimal residual (GMRES) method is used to accelerate convergence. The proposed BDDC methods allow for jumps of the coefficient across subdomain interfaces. When jumps of the coefficient appear inside subdomains, the BDDC algorithms adaptively choose the primal variables deriving from the eigenvectors of some local generalized eigenvalue problems. The adaptive BDDC algorithms with advanced deluxe scaling can ensure good performance with highly discontinuous coefficients. A convergence analysis of the BDDC method with a preconditioned GMRES iteration is provided, and several numerical experiments confirm the theoretical estimate.

Key Words.
finite volume element methods, domain decomposition, BDDC, deluxe scaling

AMS Subject Classifications.
65F10, 65N30, 65N55

Abstract.
We propose a modified alternating positive semidefinite splitting (MAPSS) preconditioner for solving block three-by-three saddle point problems that arise in linear programming and the finite element discretization of Maxwell equations. Spectral

Abstract.
The first focus of this paper is the characterization of the spectrum and the singular values of the coefficient matrix stemming from the discretization of a parabolic diffusion problem using a space-time grid and secondly from the approximation of distributed-order fractional equations. For this purpose we use the classical GLT theory and the new concept of GLT momentary symbols. The first permits us to describe the singular value or eigenvalue asymptotic distribution of the sequence of the coefficient matrices. The latter permits us to derive a function that describes the singular value or eigenvalue distribution of the matrix of the sequence, even for small matrix sizes, but under given assumptions. The paper is concluded with a list of open problems, including the use of our machinery in the study of iteration matrices, especially those concerning multigrid-type techniques.

Key Words.
Toeplitz matrices, asymptotic distribution of eigenvalues and singular values, numerical solution of discretized equations for boundary value problems involving PDEs, fractional differential equations

AMS Subject Classifications.
15B05, 34L20, 65N22, 35R11

Abstract.
We discuss a simple, easily overlooked, explicit deflation procedure applied to Golub-Kahan-Lanczos Bidiagonalization (GKLB)-based methods to compute the next set of the largest singular triplets of a matrix from an already computed partial singular value decomposition. Our results here complement the vast literature on this topic, provide additional insight, and highlight the simplicity and the effectiveness of this procedure. We demonstrate how existing GKLB-based routines for the computation of the largest singular triplets can be easily adapted to take advantage of explicit deflation, thus making it more appealing to a wider range of users. Numerical examples are presented including an application of singular value thresholding.

Key Words.
Lanczos bidiagonalization, (partial/truncated) singular value decomposition, deflation, thresholding

AMS Subject Classifications.
65F15, 65F50, 15A18

Abstract.
In this work, we propose a novel parallel-in-time preconditioner for an all-at-once system, arising from the numerical solution of linear wave equations. Namely, our main result concerns a block tridiagonal Toeplitz preconditioner that can be diagonalized via fast sine transforms, whose effectiveness is theoretically shown for the nonsymmetric block Toeplitz system resulting from discretizing the concerned wave equation. Our approach is to first transform the original linear system into a symmetric one and subsequently develop the desired preconditioning strategy based on

the spectral symbol of the modified matrix. Various Krylov subspace methods are considered. That is, we show that the minimal polynomial of the preconditioned matrix is of low degree, which leads to fast convergence when the generalized minimal residual method is used. To fully utilize the symmetry of the modified matrix, we additionally construct an absolute-value preconditioner which is symmetric positive definite. Then, we show that the eigenvalues of the preconditioned matrix are clustered around ± 1, which gives a convergence guarantee when the minimal residual method is employed. Numerical examples are given to support the effectiveness of our preconditioner. Our block Toeplitz preconditioner provides an alternative to the existing block circulant preconditioner proposed by McDonald, Pestana, and Wathen in [SIAM J. Sci. Comput., 40 (2018), pp. A1012–A1033], advancing the symmetrization preconditioning theory that originated from the same work.

Key Words.
fast sine transforms, wave equations, Krylov subspace methods, all-at-once discretization, parallel-in-time, block circulant preconditioners

AMS Subject Classifications.
15B05, 65F08, 65F10, 65M22

Abstract.

We develop a toolbox for the error analysis of linear recurrences with constant or polynomial coefficients, based on generating series, Cauchy's method of majorants, and simple results from analytic combinatorics. We illustrate the power of the approach by several nontrivial application examples. Among these examples are a new worst-case analysis of an algorithm for computing the Bernoulli numbers and a new algorithm for evaluating differentially finite functions in interval arithmetic while avoiding interval blow-up.

Key Words.
rounding error, rigorous computing, complex variable, majorant series, Bernoulli numbers, vibrating string, differentially finite function

AMS Subject Classifications.
65G50, 65Q30, 65L70, 05A15

Abstract.

Regular convergence, together with other types of convergence, have been studied since the 1970s for discrete approximations of linear operators. In this paper, we consider the eigenvalue approximation of a compact operator T that can be written as an eigenvalue problem of a holomorphic Fredholm operator function $F(\eta) = T - \frac{1}{\eta}I$. Focusing on finite element methods (conforming, discontinuous Galerkin, non-conforming, etc.), we show that the regular convergence of the discrete holomorphic operator functions F_n to F follows from the compact convergence of the discrete operators T_n to T. The convergence of the eigenvalues is then obtained using abstract approximation theory for the eigenvalue problems of holomorphic Fredholm operator functions. The result can be used to prove the convergence of various finite element methods for eigenvalue problems such as the Dirichlet eigenvalue problem and the biharmonic eigenvalue problem.

Key Words.
regular convergence, finite element methods, eigenvalue problems

AMS Subject Classifications.
65N25, 65N30

Abstract.
Cvetković-Kostić-Varga (CKV)-type matrices play a significant role in numerical linear algebra. However, verifying whether a given matrix is a CKV-type matrix is complicated because it involves choosing a suitable subset of $\{1, 2, \ldots, n\}$. In this paper, we give some easily computable and verifiable equivalent conditions for a CKV-type matrix, and based on these conditions, two direct algorithms with less computational cost for identifying CKV-type matrices are put forward. Moreover, by considering the matrix sparsity pattern, two classes of matrices called S-Sparse Ostrowski-Brauer type-I and type-II matrices are proposed and then proved to be subclasses of CKV-type matrices. The relationships with other subclasses of H-matrices are also discussed. Besides, a new eigenvalue localization set involving the sparsity pattern for matrices is presented, which requires less computational cost than that provided by Cvetković et al. [Linear Algebra Appl., 608 (2021), pp.158–184].

Key Words.
CKV-type matrices, S-Sparse Ostrowski-Brauer type-I matrices, S-Sparse Ostrowski-Brauer type-II matrices, H-matrices

AMS Subject Classifications.
15A18, 15A42, 15A69

Abstract.
In contrast to odd-length trigonometric interpolants, even-length trigonometric interpolants need not be unique; this is apparent from the representation of the interpolant in the (real or complex) Fourier basis, which possesses an extra degree of freedom in the choice of the highest-order basis function in the even case. One can eliminate this degree of freedom by imposing a constraint, but then the interpolant may cease to exist for certain choices of the interpolation points. On the other hand, the Lagrange representation developed by Gauss always produces an interpolant despite having no free parameters. We discuss the choice Gauss's formula makes for the extra degree of freedom and show that, when the points are equispaced, its choice is optimal in the sense that it minimizes both the standard and 2-norm Lebesgue constants for the interpolation problem. For non-equispaced points, we give numerical evidence that Gauss's formula is no longer optimal and consider interpolants of minimal 2-norm instead. We show how to modify Gauss's formula to produce a minimal-norm interpolant and that, if the points are equispaced, no modification is necessary.

as linear discrete ill-posed problems. Such systems arise, e.g., when discretizing a Fredholm integral equation of the first kind. The right-hand side vectors of linear discrete ill-posed problems that arise in science and engineering often represent an experimental measurement that is contaminated by measurement error. The solution to these problems typically is very sensitive to this error. Previous works have shown that error propagation into the computed solution may be reduced by using specially designed iterative methods that allow the user to select the subspace in which the approximate solution is computed. Since the dimension of this subspace often is quite small, its choice is important for the quality of the computed solution. This work describes algorithms for three iterative methods that modify the GMRES, block GMRES, and global GMRES methods for the solution of appropriate linear systems of equations. We contribute to the work already available on this topic by introducing two block variants for the solution of linear systems of equations with multiple right-hand side vectors. The dominant computational aspects are discussed, and software for each method is provided. Additionally, we illustrate the utility of these iterative subspace methods through numerical examples focusing on image reconstruction.
This paper is accompanied by software.

Key Words.
ill-posed problems, iterative method, Arnoldi process, block Arnoldi process, global Arnoldi process

AMS Subject Classifications.
65F10, 65F22

378 A numerical method for solving systems of hypersingular integro-differential equations.
Maria Carmela De Bonis, Abdelaziz Mennouni, and Donatella Occorsio.

Abstract.
This paper is concerned with a collocation-quadrature method for solving systems of Prandtl's integro-differential equations based on de la Vallée Poussin filtered interpolation at Chebyshev nodes. We prove stability and convergence in Hölder-Zygmund spaces of locally continuous functions. Some numerical tests are presented to examine the method's efficacy.

Key Words.
Chebyshev nodes, filtered approximation, Hölder-Zygmund spaces, system of Prandtl's integro-differential equations

AMS Subject Classifications.
41A10, 65D05, 33C45, 45J05

394 Conditioning of linear systems arising from penalty methods.
William Layton and Shuxian Xu.

Abstract.
Penalizing incompressibility in the Stokes problem leads, under mild assumptions, to matrices with condition numbers $\kappa = \mathcal{O}(\varepsilon^{-1}h^{-2})$, with $\varepsilon =$ penalty parameter $\ll 1$ and $h =$ meshwidth < 1. Although $\kappa = \mathcal{O}(\varepsilon^{-1}h^{-2})$ is large, practical tests seldom report difficulty in solving these systems. In the SPD case, using the conjugate

gradient method, this is usually explained by spectral gaps occurring in the penalized coefficient matrix. Herein we point out a second contributing factor. Since the solution is approximately incompressible, solution components in the eigenspaces associated with the penalty terms can be small. As a result, the *effective* condition number can be much smaller than the standard condition number.

Key Words.
penalty method, effective condition number

AMS Subject Classifications.
65F35, 15A12

402 Fast computation of Sep$_\lambda$ via interpolation-based globality certificates.
Tim Mitchell.

Abstract.
Given two square matrices A and B, we propose a new approach for computing the smallest value $\varepsilon \geq 0$ such that $A + E$ and $A + F$ share an eigenvalue, where $\|E\| = \|F\| = \varepsilon$. In 2006, Gu and Overton proposed the first algorithm for computing this quantity, called $\mathrm{sep}_\lambda(A, B)$ ("sep-lambda"), using ideas inspired from an earlier algorithm of Gu for computing the distance to uncontrollability. However, the algorithm of Gu and Overton is extremely expensive, which limits it to the tiniest of problems, and until now, no other algorithms have been known. Our new algorithm can be orders of magnitude faster and can solve problems where A and B are of moderate size. Moreover, our method consists of many "embarrassingly parallel" computations, and so it can be further accelerated on multi-core hardware. Finally, we also propose the first algorithm to compute an earlier version of sep-lambda where $\|E\| + \|F\| = \varepsilon$.

Key Words.
sep-lambda, eigenvalue separation, eigenvalue perturbation, pseudospectra, Hamiltonian matrix

AMS Subject Classifications.
15A18, 15A22, 15A42, 65F15, 65F30

432 Rational symbolic cubature rules over the first quadrant in a Cartesian plane.
Jilali Abouir, Brahim Benouahmane, and Yassine Chakir.

Abstract.
In this paper we introduce a new symbolic Gaussian formula for the evaluation of an integral over the first quadrant in a Cartesian plane, in particular with respect to the weight function $w(x) = \exp(-x^T x - 1/x^T x)$, where $x = (x_1, x_2)^T \in \mathbb{R}^2_+$. It integrates exactly a class of homogeneous Laurent polynomials with coefficients in the commutative field of rational functions in two variables. It is derived using the connection between orthogonal polynomials, two-point Padé approximants, and Gaussian cubatures. We also discuss the connection to two-point Padé-type approximants in order to establish symbolic cubature formulas of interpolatory type. Numerical examples are presented to illustrate the different formulas developed in the paper.

Key Words.
homogeneous orthogonal polynomials, homogeneous two-point Padé, symbolic Gaussian cubature

AMS Subject Classifications.
41A21, 41A20, 65D32

450 Iterative Lavrentiev regularization method under a heuristic rule for nonlinear ill-posed operator equations.
Pallavi Mahale and Ankit Singh.

Abstract.
In this paper, we consider the iterative Lavrentiev regularization method for obtaining a stable approximate solution for a nonlinear ill-posed operator equation $F(x) = y$, where $F : D(F) \subset X \to X$ is a nonlinear monotone operator on the Hilbert spaces X. In order to obtain a stable approximate solution using iterative regularization methods, it is important to use a suitable stopping rule to terminate the iterations at the appropriate step. Recently, Qinian Jin and Wei Wang (2018) have proposed a heuristic rule to stop the iterations for the iteratively regularized Gauss-Newton method. The advantage of a heuristic rule over the existing a priori and a posteriori rules is that it does not require accurate information on the noise level, which may not be available or reliable in practical applications. In this paper, we propose a heuristic stopping rule for an iterated Lavrentiev regularization method.
We derive error estimates under suitable nonlinearity conditions on the operator F.

Key Words.
Lavrentiev regularization, nonlinear ill-posed problems, heuristic parameter choice rules

AMS Subject Classifications.
65J20, 47J06

470 An enhancement of the convergence of the IDR method.
F. Bouyghf, A. Messaoudi, and H. Sadok.

Abstract.
In this paper, we consider a family of algorithms, called IDR, based on the induced dimension reduction theorem. IDR is a family of efficient short recurrence methods introduced by Sonneveld and Van Gijzen for solving large systems of nonsymmetric linear equations. These methods generate residual vectors that live in a sequence of nested subspaces. We present the IDR(s) method and give two improvements of its convergence. We also define and give a global version of the IDR(s) method and describe a partial and a complete improvement of its convergence. Moreover, we recall the block version and state its improvements. Numerical experiments are provided to illustrate the performances of the derived algorithms compared to the well-known classical GMRES method and the bi-conjugate gradient stabilized method for systems with a single right-hand side, as well as the global GMRES, the global bi-conjugate gradient stabilized, the block GMRES, and the block bi-conjugate gradient stabilized methods for systems with multiple right-hand sides.

Key Words.
linear equations, iterative methods, IDR method, Krylov subspace, global and block Krylov subspace methods

AMS Subject Classifications.
65F45, 65F

486 A computational framework for edge-preserving regularization in dynamic inverse problems.
Mirjeta Pasha, Arvind K. Saibaba, Silvia Gazzola, Malena I. Español, and Eric de Sturler.

Abstract.
We devise efficient methods for dynamic inverse problems, where both the quantities of interest and the forward operator (measurement process) may change in time. Our goal is to solve for all the quantities of interest simultaneously. We consider large-scale ill-posed problems made more challenging by their dynamic nature and, possibly, by the limited amount of available data per measurement step. To alleviate these difficulties, we apply a unified class of regularization methods that enforce simultaneous regularization in space and time (such as edge enhancement at each time instant and proximity at consecutive time instants) and achieve this with low computational cost and enhanced accuracy. More precisely, we develop iterative methods based on a majorization-minimization (MM) strategy with quadratic tangent majorant, which allows the resulting least-squares problem with a total variation regularization term to be solved with a generalized Krylov subspace (GKS) method; the regularization parameter can be determined automatically and efficiently at each iteration. Numerical examples from a wide range of applications, such as limited-angle computerized tomography (CT), space-time image deblurring, and photoacoustic tomography (PAT), illustrate the effectiveness of the described approaches.

Key Words.
dynamic inversion, time-dependence, edge-preservation, majorization-minimization, regularization, generalized Krylov subspaces, image deblurring, photoacoustic tomography, computerized tomography

AMS Subject Classifications.
65F10, 65F22, 65F50

517 A Gauss-Laguerre approach for the resolvent of fractional powers.
Eleonora Denich, Laura Grazia Dolce, and Paolo Novati.

Abstract.
This paper introduces a very fast method for the computation of the resolvent of fractional powers of operators. The analysis is kept in the continuous setting of (potentially unbounded) self-adjoint positive operators in Hilbert spaces. The method is based on the Gauss-Laguerre rule, exploiting a particular integral representation of the resolvent. We provide sharp error estimates that can be used to a priori select the number of nodes to achieve a prescribed tolerance.

Key Words.
resolvent of fractional powers, Gauss-Laguerre rule, functions of operators

AMS Subject Classifications.
47A58, 65F60, 65D32

538 Inexact rational Krylov subspace methods for approximating the action of functions of matrices.
Shengjie Xu and Fei Xue.

Abstract.
This paper concerns the theory and development of inexact rational Krylov subspace methods for approximating the action of a function of a matrix $f(A)$ to a column vector b. At each step of the rational Krylov subspace methods, a shifted linear system of equations needs to be solved to enlarge the subspace. For large-scale problems, such a linear system is usually solved approximately by an iterative method. The main question is how to relax the accuracy of these linear solves without negatively affecting the convergence of the approximation of $f(A)b$. Our insight into this issue is obtained by exploring the residual bounds for the rational Krylov subspace approximations of $f(A)b$, based on the decaying behavior of the entries in the first column of certain matrices of A restricted to the rational Krylov subspaces. The decay bounds for these entries for both analytic functions and Markov functions can be efficiently and accurately evaluated by appropriate quadrature rules. A heuristic based on these bounds is proposed to relax the tolerances of the linear solves arising in each step of the rational Krylov subspace methods. As the algorithm progresses toward convergence, the linear solves can be performed with increasingly lower accuracy and computational cost. Numerical experiments for large nonsymmetric matrices show the effectiveness of the tolerance relaxation strategy for the inexact linear solves of rational Krylov subspace methods.

Key Words.
matrix functions, rational Krylov subspace, inexact Arnoldi algorithm, decay bounds

AMS Subject Classifications.
65D15, 65F10, 65F50, 65F60

568 An evolutionary approach to the coefficient problems in the class of starlike functions.
Piotr Jastrzębski and Adam Lecko.

Abstract.
In this paper, we apply the differential evolution algorithm as a new approach to solve some coefficient problems within Geometric Function Theory. We find sharp bounds for the determinant of the Hankel matrix $H_{4,1}(f)$ and the determinants of all its sub-matrices for the class of starlike functions, i.e., for the class of all analytic injective functions f in the unit disk $\mathbb{D} := \{z \in \mathbb{C} : |z| < 1\}$ normalized by $f(0) = f'(0) - 1 = 0$ such that $f(\mathbb{D})$ is a starlike set with respect to the origin. In addition, a relevant conjecture regarding some coefficient functionals related to the Zalcman functional is formulated.

Key Words.
differential evolution algorithm, Hankel determinant, starlike function, Carathéodory class and Zalcman functional

AMS Subject Classifications.
65K05, 30C45, 30C50

582 An optimal method for recovering the mixed derivative $\boldsymbol{f^{(2,2)}}$ of bivariate functions.
Y. V. Semenova and S. G. Solodky.

Abstract.
The problem of recovering the mixed derivative $\boldsymbol{f^{(2,2)}}$ for bivariate functions is

investigated. Based on the truncation method, a numerical differentiation algorithm is constructed that uses perturbed Fourier–Legendre coefficients of the function as input information. Moreover, the idea of a hyperbolic cross is implemented, which makes it possible to significantly reduce computational costs. It is established that this algorithm guarantees order-optimal accuracy (in the power scale) with a minimal amount of Galerkin information involved.

Key Words.
numerical differentiation, Legendre polynomials, truncation method, information complexity, optimal error estimates

AMS Subject Classifications.
47A52, 65D25

597 Convergence rates of individual Ritz values in block preconditioned gradient-type eigensolvers.
Ming Zhou and Klaus Neymeyr.

Abstract.
Many popular eigensolvers for large and sparse Hermitian matrices or matrix pairs can be interpreted as accelerated block preconditioned gradient (BPG) iterations for the purpose of analyzing their convergence behavior by composing known estimates. An important feature of the BPG method is the cluster robustness, i.e., that reasonable performance for computing clustered eigenvalues is ensured by a sufficiently large block size. Concise estimates reflecting this feature can easily be derived for exact-inverse (exact shift-inverse) preconditioning. Therein, the BPG method is compatible with an abstract block iteration analyzed by Knyazev [Soviet J. Numer. Anal. Math. Modelling, 2 (1987), pp. 371–396]. An adaptation to more general preconditioning is difficult as some orthogonality properties cannot be preserved. Another analysis by Ovtchinnikov [Linear Algebra Appl., 415 (2006), pp. 140–166] provides sumwise estimates for Ritz values containing elegant convergence factors. However, additional technical terms lead to cumbersome bounds and could cause overestimations in the first steps. We expect to improve the existing results by deriving concise estimates for individual Ritz values. A mid-term goal has been achieved for the BPG iteration with fixed step sizes by the authors in [Math. Comp., 88 (2019), pp. 2737–2765]. The present paper deals with the more practical case that the step sizes are implicitly optimized by the Rayleigh–Ritz method.

Key Words.
preconditioned subspace eigensolvers, Ritz values, cluster robustness

AMS Subject Classifications.
65F15, 65N12, 65N25

621 A generalized Sassenfeld criterion and its relation to H-matrices.
Thomas P. Wihler.

Abstract.
The starting point of this note is a decades-old yet little-noticed sufficient condition, presented by Sassenfeld in 1951, for the convergence of the classical Gauß–Seidel method. The purpose of the present paper is to shed new light on Sassenfeld's criterion and to demonstrate that it is closely related to H-matrices. In particular, our

main result yields a novel characterization of H-matrices. In addition, a new convergence estimate for iterative linear solvers, which involve H-matrix preconditioners, is briefly discussed.

Key Words.
Sassenfeld criterion, convergence of iterative linear solvers, splitting methods, Gauß–Seidel scheme, preconditioning, H-matrices

AMS Subject Classifications.
15B48, 65F08, 65F10

629 Preconditioned Chebyshev BiCG method for parameterized linear systems.
Siobhán Correnty, Elias Jarlebring, and Daniel B. Szyld.

Abstract.
We consider the problem of approximating the solution to $\boldsymbol{A}(\boldsymbol{\mu})\boldsymbol{x}(\boldsymbol{\mu}) = \boldsymbol{b}$ for many different values of the parameter $\boldsymbol{\mu}$. Here, $\boldsymbol{A}(\boldsymbol{\mu})$ is large, sparse, and nonsingular with a nonlinear dependence on $\boldsymbol{\mu}$. Our method is based on a companion linearization derived from an accurate Chebyshev interpolation of $\boldsymbol{A}(\boldsymbol{\mu})$ on the interval $[-\boldsymbol{a}, \boldsymbol{a}]$, $\boldsymbol{a} \in \mathbb{R}_+$, inspired by Effenberger and Kressner [BIT, 52 (2012), pp. 933–951]. The solution to the linearization is approximated in a preconditioned BiCG setting for shifted systems, as proposed in Ahmad et al. [SIAM J. Matrix Anal. Appl., 38 (2017), pp. 401–424], where the Krylov basis matrix is formed once. This process leads to a short-term recurrence method, where one execution of the algorithm produces the approximation of $\boldsymbol{x}(\boldsymbol{\mu})$ for many different values of the parameter $\boldsymbol{\mu} \in [-\boldsymbol{a}, \boldsymbol{a}]$ simultaneously. In particular, this work proposes one algorithm which applies a shift-and-invert preconditioner exactly as well as an algorithm which applies the preconditioner inexactly based on the work by Vogel [Appl. Math. Comput., 188 (2007), pp. 226–233]. The competitiveness of the algorithms is illustrated with large-scale problems arising from a finite element discretization of a Helmholtz equation with a parameterized material coefficient. The software used in the simulations is publicly available online, and thus all our experiments are reproducible.

Key Words.
parameterized linear systems, short-term recurrence methods, Chebyshev interpolation, inexact preconditioning, Krylov subspace methods, companion linearization, shifted linear systems, parameterized Helmholtz equation, time-delay systems

AMS Subject Classifications.
15A06, 65F08, 65F10, 65F50, 65N22, 65P99

A1 Reuben Louis Rosenberg (1909–1986) and the Stein–Rosenberg theorem.
Claude Brezinski and Michela Redivo-Zaglia.

Abstract.
Rosenberg is well known by numerical analysts, in particular by those working on numerical linear algebra, for an important theorem on the convergence of the methods of Jacobi and Gauss–Seidel for solving systems of linear equations. This paper was published in 1948 with Philip Stein (1890–1974). Although the biography of Stein is well known, that of Rosenberg was almost completely unknown. This paper presents a complete biography of Reuben Louis Rosenberg (1909–1986) with an analysis of all his scientific contributions in numerical analysis and as well as

in nuclear physics. Personal information are also included. The Stein–Rosenberg theorem is commented, replaced in its historical context, and a large review of its variants, generalizations, and applications is given.

Key Words.
Reuben Louis Rosenberg, relaxation methods, Stein–Rosenberg theorem, numerical linear algebra, biography, nuclear physics

AMS Subject Classifications.
01A60, 01A70, 65F10, 81V35, 82C40