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Contents

Abstract.
We consider compact composite linear operators in Hilbert space, where the composition is given by some compact operator followed by some non-compact one possessing a non-closed range. Focus is on the impact of the non-compact factor on the overall behavior of the decay rates of the singular values of the composition. Specifically, the composition of the compact integration operator with the non-compact Hausdorff moment operator is considered. We show that the singular values of the composite operator decay faster than those of the integration operator, providing a first example of this kind. However, there is a gap between available lower bounds for the decay rate and the obtained result. Therefore we conclude with a discussion.

Key Words.
Hausdorff moment problem, linear inverse problem, degree of ill-posedness, composite operator, conditional stability

AMS Subject Classifications.
47A52, 47B06, 65J20, 44A60

Abstract.
We state some sufficient criteria for the tangential cone conditions to hold for the electrical impedance tomography problem. The results are based on Löwner convexity of the forward operator. As a consequence, we show that for conductivities that satisfy various properties, such as Hölder source conditions, finite-dimensionality, or certain monotonicity criteria, the tangential cone condition is verified.

Key Words.
impedance tomography, tangential cone condition, Löwner convexity

AMS Subject Classifications.
47J06, 65F22

Abstract.
In this note we consider spectral cut-off estimators to solve a statistical linear inverse problem under arbitrary white noise. The truncation level is determined with a recently introduced adaptive method based on the classical discrepancy principle. We

provide probabilistic oracle inequalities together with quantification of uncertainty for general linear problems. Moreover, we compare the new method to existing ones, namely the early stopping sequential discrepancy principle and the balancing principle, both theoretically and numerically.

Key Words.
statistical inverse problems, non-Bayesian approach, discrepancy principle, oracle inequality, early stopping

AMS Subject Classifications.
65J20, 62G99

57 A note on numerical singular values of compositions with non-compact operators.
Daniel Gerth.

Abstract.
Linear non-compact operators are difficult to study because they do not exist in the finite-dimensional world. Recently, Hofmann and Mathé [Electron. Trans. Numer. Anal., 57 (2022), pp. 1–16] studied the singular values of the compact composition of the non-compact Hausdorff moment operator and the compact integral operator and found credible arguments, but no strict proof, that those singular values fall only slightly faster than those of the integral operator alone. However, the fact that numerically the singular values of the combined operator fall exponentially fast was not mentioned. In this note, we supply the missing numerical results and provide an explanation why the two seemingly contradictory results may both be true.

Key Words.
singular values, ill-posedness, compact operator, non-compact operator

AMS Subject Classifications.
15A18, 65F35

67 The Levenberg–Marquardt regularization for the backward heat equation with fractional derivative.
Pornsarp Pornsawad, Christine Böckmann, and Wannapa Panitsupakamon.

Abstract.
The backward heat problem with time-fractional derivative in Caputo's sense is studied. The inverse problem is severely ill-posed in the case when the fractional order is close to unity. A Levenberg–Marquardt method with a new a posteriori stopping rule is investigated. We show that optimal order can be obtained for the proposed method under a Hölder-type source condition. Numerical examples for one and two dimensions are provided.

Key Words.
ill-posed problems, time-fractional derivative, backward heat problem, Levenberg–Marquardt method, a posteriori stopping rule, optimal order

AMS Subject Classifications.
26A33, 47A52, 65R30, 65M30

Abstract.
We consider a general setting for dynamic tensor field tomography in an inhomogeneous refracting and absorbing medium as an inverse source problem for the associated transport equation. Following Fermat's principle, the Riemannian metric in the considered domain is generated by the refractive index of the medium. There is a wealth of results for the inverse problem of recovering a tensor field from its longitudinal ray transform in a static Euclidean setting, whereas there are only a few inversion formulas and algorithms existing for general Riemannian metrics and time-dependent tensor fields. It is a well-known fact that tensor field tomography is equivalent to an inverse source problem for a transport equation where the ray transform serves as given boundary data. We prove that this result extends to the dynamic case. Interpreting dynamic tensor tomography as an inverse source problem represents a holistic approach in this field. To guarantee that the forward mappings are well defined, it is necessary to prove existence and uniqueness for the underlying transport equations. Unfortunately, the bilinear forms of the associated weak formulations do not satisfy the coercivity condition. To this end we transfer to viscosity solutions and prove their unique existence in appropriate Sobolev (static case) and Sobolev–Bochner (dynamic case) spaces under a certain assumption that allows only small variations of the refractive index. Numerical evidence is given that the viscosity solution solves the original transport equation if the viscosity term turns to zero.

Key Words.
attenuated refractive dynamic ray transform of tensor fields, geodesics, transport equation, viscosity solutions

AMS Subject Classifications.
35F10, 35F16, 45Q05

Abstract.
This paper studies Tikhonov regularization for finitely smoothing operators in Banach spaces when the penalization enforces too much smoothness in the sense that the penalty term is not finite at the true solution. In a Hilbert space setting, Natterer [Applicable Anal., 18 (1984), pp. 29–37] showed with the help of spectral theory that optimal rates can be achieved in this situation. ("Oversmoothing does not harm.") For oversmoothing variational regularization in Banach spaces, only very recently has progress been achieved in several papers in different settings, all of which construct families of smooth approximations to the true solution. In this paper we propose to construct such a family of smooth approximations based on K-interpolation theory. We demonstrate that this leads to simple, self-contained proofs and to rather general results. In particular, we obtain optimal convergence rates for bounded variation regularization, general Besov penalty terms, and ℓ^p wavelet penalization with $p < 1$, which cannot be treated by previous approaches. We also derive minimax optimal rates for white noise models. Our theoretical results are confirmed in numerical experiments.

Key Words.
regularization, convergence rates, oversmoothing, BV-regularization, sparsity-promoting wavelet regularization, statistical inverse problems

AMS Subject Classifications.
65J22, 65N21, 35R20

Abstract.
This paper is concerned with variational regularization of inverse problems where both the data and the forward operator are given only approximately. We propose a general approach to derive error estimates which separates the analysis of smoothness of the exact solution from the analysis of the effect of errors in the data and the operator. Our abstract error bounds are applied to both discrete and continuous data, random and deterministic types of error, as well as Huber data fidelity terms for impulsive noise.

Key Words.
inverse problem, variational regularization, error bounds, operator noise, random noise

AMS Subject Classifications.
65J20, 60H30, 60H40

Abstract.
Reconstruction of the neuronal current inside the human brain from non-invasive measurements of the magnetic flux density via magnetoencephalography (MEG) or of electric potential differences via electroencephalography (EEG) is an invaluable tool for neuroscientific research, as it provides measures of activity in the brain. However, it is also a severely ill-posed inverse problem. Assuming spherical geometries, we consider the spherical multiple-shell model for the inverse MEG and EEG problem and apply the regularized functional matching pursuit algorithm (RFMP) for its solution. We present a new convergence proof for the RFMP for operators between two infinite-dimensional Hilbert spaces. Moreover, we utilize the complementarity of EEG and MEG data to combine inversions of simultaneous electric and magnetic measurements. Finally, we test the algorithm numerically on synthetic data using several Sobolev norms as penalty term and apply it to real data.

Key Words.
electroencephalography, greedy algorithms, ill-posed problems, integral equation, inverse problems, magnetoencephalography, regularization methods, regularized functional matching pursuit, Sobolev spaces

AMS Subject Classifications.
42C10, 45B05, 46C07, 46N40, 47A52, 65R30, 65R32

Abstract.
We present a framework enabling variational data assimilation for gradient flows in general metric spaces, based on the minimizing movement (or Jordan–Kinderlehrer–Otto) approximation scheme. After discussing stability properties in the most general case, we specialize to the space of probability measures endowed with the Wasserstein distance. This setting covers many non-linear partial differential equations (PDEs), such as the porous-medium equation or general drift–diffusion–aggregation equations, which can be treated by our methods independently of their respective properties (such as finite speed of propagation or blow-up). We then focus on the numerical implementation using a primal–dual algorithm. The strength of our approach lies in the fact that, by simply changing the driving functional, a wide range of PDEs can be treated without the need to adopt the numerical scheme. We conclude by presenting several numerical examples.

Key Words.
gradient flows, variational data assimilation, primal–dual algorithms, Wasserstein distance

AMS Subject Classifications.
49N45, 35R30, 49N45, 49Q22

Abstract.
The choice of a suitable regularization parameter is an important part of most regularization methods for inverse problems. In the absence of reliable estimates of the noise level, heuristic parameter choice rules can be used to accomplish this task. While they are already fairly well understood and tested in the case of linear problems, not much is known about their behaviour for nonlinear problems and even less in the respective case of iterative regularization. Hence, in this paper, we numerically study the performance of some of these rules when used to determine a stopping index for Landweber iteration for various nonlinear inverse problems. These are chosen from different practically relevant fields such as integral equations, parameter estimation, and tomography.

Key Words.
heuristic parameter choice rules, Landweber iteration, inverse and ill-posed problems, nonlinear operator equations, numerical comparison

AMS Subject Classifications.
65J20, 65F22, 47J06, 35R25