

Contents

- 1** Isogeometric analysis for singularly perturbed problems in 1-D: error estimates.
Christos Xenophontos and Irene Sykopetritou.
- Abstract.**
We consider one-dimensional singularly perturbed boundary value problems of reaction-convection-diffusion type, and the approximation of their solution using isogeometric analysis. In particular, we use a Galerkin formulation with B-splines as basis functions, defined on appropriately chosen knot vectors. We prove robust exponential convergence in the energy norm, independently of the singular perturbation parameters, and illustrate our findings through a numerical example.
- Key Words.**
singularly perturbed problem, reaction-convection-diffusion, boundary layers, isogeometric analysis, robust exponential convergence
- AMS Subject Classifications.**
65N30
- 26** A block J -Lanczos method for Hamiltonian matrices.
Atika Archid, Abdeslem Hafid Bentbib, and Said Agoujl.
- Abstract.**
This work aims to present a structure-preserving block Lanczos-like method. The Lanczos-like algorithm is an effective way to solve large sparse Hamiltonian eigenvalue problems. It can also be used to approximate $\exp(A)V$ for a given large square matrix A and a tall-and-skinny matrix V such that the geometric property of V is preserved, which interests us in this paper. This approximation is important for solving systems of ordinary differential equations (ODEs) or time-dependent partial differential equations (PDEs). Our approach is based on a block J -tridiagonalization procedure of a Hamiltonian and skew-symmetric matrix using symplectic similarity transformations.
- Key Words.**
block J -Lanczos method, Hamiltonian matrix, skew-Hamiltonian matrix, symplectic matrix, symplectic reflector, block J -tridiagonal form, block J -Hessenberg form
- AMS Subject Classifications.**
65F15, 65F30, 65F50
- 43** Coarse spaces for FETI-DP and BDDC Methods for heterogeneous problems: connections of deflation and a generalized transformation-of-basis approach.
Axel Klawonn, Martin Kühn, and Oliver Rheinbach.
- Abstract.**
In FETI-DP (Finite Element Tearing and Interconnecting) and BDDC (Balancing

Domain Decomposition by Constraints) domain decomposition methods, the convergence behavior of the iterative scheme can be improved by implementing a coarse space using a transformation of basis and local assembly. This is an alternative to coarse spaces implemented by deflation or balancing. The transformation-of-basis approaches are more robust with respect to inexact solvers than deflation and therefore more suitable for multilevel extensions. In this paper, we show a correspondence of FETI-DP or BDDC methods using a generalized transformation-of-basis approach and of FETI-DP methods using deflation or balancing, where the deflation vectors are obtained from the transformation of basis. These methods then have essentially the same eigenvalues. As opposed to existing theory, this result also applies to general scalings and highly heterogeneous problems.

We note that the new methods differ slightly from the classic FETI-DP and BDDC methods using a transformation of basis and that the classic theory has to be replaced. An important application for the theory presented in this paper are FETI-DP and BDDC methods with adaptive coarse spaces, i.e., where deflation vectors are obtained from approximating local eigenvectors. These methods have recently gained considerable interest.

Key Words.

domain decomposition, FETI-DP, BDDC, coarse space, deflation, transformation of basis, change of variables, elliptic partial differential equations

AMS Subject Classifications.

65N30,65N25,65N50,65N55,74E05

- 77 Leap-frog method for stochastic functional wave equations.
Henryk Leszczyński, Milena Matusik, and Monika Wrzosek.

Abstract.

We perform a time-space discretisation, known as the leap-frog method, for nonlinear stochastic functional wave equations driven by multiplicative time-space white noise. To prove its stability we apply Cairoli's maximal inequalities for two-parameter martingales and provide a lemma for estimating solutions to a class of stochastic wave equations and a Gronwall-type inequality over cones. The method converges in L^2 at a rate of $O(\sqrt{h})$, where h is a time-space step size.

Key Words.

leap-frog, stochastic wave equation

AMS Subject Classifications.

60H15, 35R60, 35R10

- 88 The swallowtail integral in the highly oscillatory region II.
Chelo Ferreira, José L. López, and Ester Pérez Sinusía.

Abstract.

We analyze the asymptotic behavior of the swallowtail integral $\int_{-\infty}^{\infty} e^{i(t^5 + xt^3 + yt^2 + zt)} dt$ for large values of $|y|$ and bounded values of $|x|$ and $|z|$. We use the simplified saddle point method introduced in [López et al., J. Math. Anal. Appl., 354 (2009), pp. 347–359]. With this method, the analysis is more straightforward than with the standard saddle point method, and it is possible to derive complete asymptotic expansions of the integral for large $|y|$ and fixed x and z . There are four Stokes lines in the sector $(-\pi, \pi]$ that divide the complex

y -plane into four sectors in which the swallowtail integral behaves differently when $|y|$ is large. The asymptotic approximation is the sum of two asymptotic series whose terms are elementary functions of x , y , and z . One of them is of Poincaré type and is given in terms of inverse powers of $y^{1/2}$. The other one is given in terms of an asymptotic sequence whose terms are of the order of inverse powers of $y^{1/9}$ when $|y| \rightarrow \infty$, and it is multiplied by an exponential factor that behaves differently in the four mentioned sectors. Some numerical experiments illustrate the accuracy of the approximation.

Key Words.

swallowtail integral, asymptotic expansions, modified saddle point method

AMS Subject Classifications.

33E20, 41A60

- 100** An empirical study of transboundary air pollution of the Beijing-Tianjin region.
Zuliang Lu, Fei Huang, Lin Li, Xiaoxiao Zuo, and Junhong Li.

Abstract.

In this paper, we present a stochastic differential game to model the transboundary air pollution problems of the Beijing-Tianjin region with emission permits trading. By using stochastic optimal control theory, we obtain the Hamilton-Jacobi-Bellman equation satisfied by the value function for the cooperative games. Next, we solve the Hamilton-Jacobi-Bellman equation by using a fitted finite volume method. Finally, the efficiency and the usefulness of the fitted finite volume method are illustrated by an empirical study.

Key Words.

transboundary air pollution of the Beijing-Tianjin region, stochastic differential game, emission permits trading, Hamilton-Jacobi-Bellman equation, fitted finite volume method

AMS Subject Classifications.

49J20, 65N30

- 113** Fractional Hermite interpolation for non-smooth functions.
Jiayin Zhai, Zhiyue Zhang, and Tongke Wang.

Abstract.

The interpolation of functions plays a fundamental role in numerical analysis. The highly accurate approximation of non-smooth functions is a challenge in science and engineering as traditional polynomial interpolation cannot characterize the singular features of these functions. This paper aims at designing a fractional Hermite interpolation for non-smooth functions based on the local fractional Taylor expansion and at deriving the corresponding explicit formula and its error remainder. We also present a piecewise hybrid Hermite interpolation scheme, a combination of fractional Hermite interpolation and traditional Hermite interpolation. Some numerical examples are presented to show the high accuracy of the fractional Hermite interpolation method.

Key Words.

non-smooth function, local fractional Taylor expansion, fractional Hermite interpolation, error remainder

AMS Subject Classifications.

26A30, 41A05, 65D05, 97N50

- 132** A spectral Newton-Schur algorithm for the solution of symmetric generalized eigenvalue problems.

Vassilis Kalantzis.

Abstract.

This paper proposes a numerical algorithm based on spectral Schur complements to compute a few eigenvalues and the associated eigenvectors of symmetric matrix pencils. The proposed scheme follows an algebraic domain decomposition viewpoint and transforms the generalized eigenvalue problem into one of computing roots of scalar functions. These scalar functions are defined so that their roots are equal to the eigenvalues of the original pencil, and these roots are computed by Newton's method. We describe the theoretical aspects of the proposed scheme and demonstrate its performance on a few test problems.

Key Words.

spectral Schur complements, domain decomposition, symmetric generalized eigenvalue problem, Newton's method

AMS Subject Classifications.

65F15, 15A18, 65F50

- 154** Coercive space-time finite element methods for initial boundary value problems.

Olaf Steinbach and Marco Zank.

Abstract.

We propose and analyse new space-time Galerkin-Bubnov-type finite element formulations of parabolic and hyperbolic second-order partial differential equations in finite time intervals. Using Hilbert-type transformations, this approach is based on elliptic reformulations of first- and second-order time derivatives, for which the Galerkin finite element discretisation results in positive definite and symmetric matrices. For the variational formulation of the heat and wave equations, we prove related stability conditions in appropriate norms, and we discuss the stability of related finite element discretisations. Numerical results are given which confirm the theoretical results.

Key Words.

space-time FEM, heat equation, wave equation

AMS Subject Classifications.

65M60

- 195** Incomplete beta polynomials.

Manuel Bello-Hernández.

Abstract.

We study some properties of incomplete beta polynomials, in particular, their zero asymptotic distribution. These polynomials satisfy a three-term recurrence relation, so they can be written as the sum of two terms each one leading to their asymptotic behavior in a region.

Key Words.

incomplete beta function, zero distribution, asymptotics, recurrence relations, potential theory

AMS Subject Classifications.

33B20, 41A58, 41A80

- 203** The series expansions and Gauss-Legendre rule for computing arbitrary derivatives of the Beta-type functions.

Junlin Li, Tongke Wang, and Yonghong Hao.

Abstract.

The beta-type functions play an important role in many applied sciences. The partial derivatives of the beta function and the incomplete beta function are integrals involving algebraic and logarithmic endpoint singularities. In this paper, some series expansions for these beta-type functions are found, which are easily used to evaluate these functions with prescribed precision. On the other hand, an accurate Gauss-Legendre quadrature formula is designed to compute these beta-type functions and their partial derivatives based on the Puiseux series for the integrands at their singularities. Some numerical examples confirm the high accuracy and high efficiency of the two algorithms, and also show that the algorithms can be used to effectively evaluate the generalized beta-type functions.

Key Words.

beta-type functions, higher-order partial derivatives, series expansion, modified Gauss-Legendre rule

AMS Subject Classifications.

65D20, 33B15, 33B20

- 214** ADMM-Softmax: an ADMM approach for multinomial logistic regression.

Samy Wu Fung, Sanna Tyrväinen, Lars Ruthotto, and Eldad Haber.

Abstract.

We present ADMM-Softmax, an alternating direction method of multipliers (ADMM) for solving multinomial logistic regression (MLR) problems. Our method is geared toward supervised classification tasks with many examples and features. It decouples the nonlinear optimization problem in MLR into three steps that can be solved efficiently. In particular, each iteration of ADMM-Softmax consists of a linear least-squares problem, a set of independent small-scale smooth, convex problems, and a trivial dual variable update. The solution of the least-squares problem can be accelerated by pre-computing a factorization or preconditioner, and the separability in the smooth, convex problem can be easily parallelized across examples. For two image classification problems, we demonstrate that ADMM-Softmax leads to improved generalization compared to a Newton-Krylov, a quasi Newton, and a stochastic gradient descent method.

Key Words.

machine learning, nonlinear optimization, alternating direction method of multipliers, classification, multinomial regression

AMS Subject Classifications.

65J22, 90C25, 49M27

- 230** A boundary and finite element coupling for a magnetically nonlinear eddy current problem.

Ramiro Acevedo, Edgardo Alvarez, and Paulo Navia.

Abstract.

The aim of this paper is to provide a mathematical and numerical analysis for a FEM-BEM coupling approximation of a magnetically nonlinear eddy current formulation by using FEM only on the conducting domain, and by imposing the integral conditions on its boundary. The nonlinear relationship between flux density and the magnetic field intensity is given by a physical parameter called magnetic reluctivity, which is assumed to depend on the Euclidean norm of the magnetic induction in the conducting domain. We use the nonlinear monotone operator theory for parabolic equations to show that the continuous formulation obtained for the coupling is a well-posed problem. Furthermore, we use Nédélec edge elements, standard nodal finite elements, and a backward-Euler time scheme, to obtain a fully discrete formulation and to prove quasi-optimal error estimates.

Key Words.

Time-dependent electromagnetic, eddy current model, nonlinear problems, boundary element method, finite element method.

AMS Subject Classifications.

65M60, 65M38, 78M10, 78M15.

- 249** Approximation of Gaussians by spherical Gauss-Laguerre basis in the weighted Hilbert space.

Nadiia Derevianko and Jürgen Prestin.

Abstract.

This paper is devoted to the study of approximation of Gaussian functions by their partial Fourier sums of degree $N \in \mathbb{N}$ with respect to the spherical Gauss-Laguerre (SGL) basis in the weighted Hilbert space $L_2(\mathbb{R}^3, \omega_\lambda)$, where $\omega_\lambda(|\mathbf{x}|) = \exp(-|\mathbf{x}|^2/\lambda)$, $\lambda > 0$. We investigate the behavior of the corresponding error of approximation with respect to the scale factor λ and order of expansion N . As interim results we obtained formulas for the Fourier coefficients of Gaussians with respect to SGL basis in the space $L_2(\mathbb{R}^3, \omega_\lambda)$. Possible application of obtained results to the docking problem are described.

Key Words.

spherical harmonic, Laguerre polynomial, Gaussian, hypergeometric function, molecular docking

AMS Subject Classifications.

33C05, 33C45, 33C55, 42C10

- 270** Asymptotic inversion of the binomial and negative binomial cumulative distribution functions.

A. Gil, J. Segura, and N. M. Temme.

Abstract.

The computation and inversion of the binomial and negative binomial cumulative distribution functions play a key role in many applications. In this paper, we explain how methods used for the central beta distribution function (described in Gil,

Segura, and Temme, [Numer. Algorithms, 74 (2017), pp. 77–91]) can be utilized to obtain asymptotic representations of these functions and also for their inversion. The performance of the asymptotic inversion methods is illustrated with numerical examples.

Key Words.

binomial cumulative distribution function, negative binomial cumulative distribution function, asymptotic representation, asymptotic inversion methods

AMS Subject Classifications.

33B20, 41A60

281 Pressure-robustness in quasi-optimal a priori estimates for the Stokes problem.

Alexander Linke, Christian Merdon, and Michael Neilan.

Abstract.

Recent analysis of the divergence constraint in the incompressible Stokes/Navier-Stokes problem has stressed the importance of equivalence classes of forces and how they play a fundamental role for an accurate space discretization. Two forces in the momentum balance are velocity-equivalent if they lead to the same velocity solution, i.e., if and only if the forces differ by only a gradient field. Pressure-robust space discretizations are designed to respect these equivalence classes. One way to achieve pressure-robust schemes is to introduce a non-standard discretization of the right-hand side forcing term for any inf-sup stable mixed finite element method. This modification leads to pressure-robust and optimal-order discretizations, but a proof was only available for smooth situations and remained open in the case of minimal regularity, where it cannot be assumed that the vector Laplacian of the velocity is at least square-integrable. This contribution closes this gap by delivering a general estimate for the consistency error that depends only on the regularity of the data term. Pressure-robustness of the estimate is achieved by the fact that the new estimate only depends on the L^2 -norm of the Helmholtz-Hodge projector of the data term and not on the L^2 -norm of the entire data term. Numerical examples illustrate the theory.

Key Words.

incompressible Stokes equations, mixed finite elements methods, a-priori error estimates, stability estimates, pressure-robustness

AMS Subject Classifications.

65N12, 65N30, 76D07

295 Chebyshev spectral collocation in space and time for the heat equation.

S. H. Lui and Sarah Nataj.

Abstract.

Spectral methods can solve elliptic partial differential equations (PDEs) numerically with errors bounded by an exponentially decaying function of the number of modes when the solution is analytic. For time-dependent problems, almost all focus has been on low-order finite difference schemes for the time derivative and spectral schemes for the spatial derivatives. This mismatch destroys the spectral convergence of the numerical solution. Spectral methods that converge spectrally in both space and time have appeared recently. This paper shows that a Chebyshev spectral collocation method of Tang and Xu for the heat equation converges exponentially when

the solution is analytic. We also derive a condition number estimate of the global spectral operator. Another space-time Chebyshev collocation scheme that is easier to implement is proposed and analyzed. This paper is a continuation of the first author's earlier paper in which two Legendre space-time collocation methods were analyzed.

Key Words.

spectral collocation, Chebyshev collocation, space-time, time-dependent partial differential equation

AMS Subject Classifications.

65M70, 65L05, 35K20, 35L20, 41A10

- 320** Error analysis for regularized multidimensional sampling expansions.

Rashad M. Asharabi and Fatemah M. Al-Abbas.

Abstract.

As it is known, the convergence rate of the multidimensional Whittaker-Kotelnikov-Shannon (WKS) sampling series is slow due to the slow decay of the sinc function. In this paper, we incorporate a convergence factor from the Bernstein space into the multidimensional WKS sampling series to establish regularized sampling and a corresponding improved convergence rate. The convergence rate of this regularized series depends on the decay of the convergence factor. Various bounds for the truncation of the regularized sampling series are investigated depending on the convergence factor. Furthermore, we estimate two types of perturbation errors associated with this series. Some numerical experiments are presented.

Key Words.

multidimensional sampling, error analysis, convergence rate

AMS Subject Classifications.

41A80, 32A05, 41A25, 41A30, 94A20

- 342** Rush-Larsen time-stepping methods of high order for stiff problems in cardiac electrophysiology.

Yves Coudière, Charlie Douanla Lontsi, and Charles Pierre.

Abstract.

The stability and accuracy of numerical methods for reaction-diffusion equations still need improvements, which prompts the development of high-order and stable time-stepping methods. This is particularly true in the context of cardiac electrophysiology, where reaction-diffusion equations are coupled with stiff systems of ordinary differential equations. So as to address these issues, much research on implicit-explicit methods and exponential integrators has been carried out during the past 15 years. In 2009, Perego and Veneziani [Electron. Trans. Numer. Anal., 35 (2009), pp. 234–256] proposed an innovative time-stepping scheme of order 2. In this paper we present an extension of this scheme to the orders 3 and 4, which we call Rush-Larsen schemes of order k . These new schemes are explicit multi-step methods, which belong to the classical class of exponential integrators. Their general formulation is simple and easy to implement. We prove that they are stable under perturbation and convergent of order k . We analyze their Dahlquist stability and show that they have a very large stability domain provided that the stabilizer associated with the method captures well enough the stiff modes of the problem. We

study their application to a system of equations that models the action potential in cardiac electrophysiology.

Key Words.

stiff equations, explicit high-order multistep methods, exponential integrators, stability and convergence, Dahlquist stability

AMS Subject Classifications.

65L04, 65L06, 65L20, 65L99

- 358** On recurrences converging to the wrong limit in finite precision and some new examples.

Siegfried M. Rump.

Abstract.

In 1989, Jean-Michel Muller gave a famous example of a recurrence where, for particular initial values, the iteration over real numbers converges to a repellent fixed point, whereas finite precision arithmetic produces a different result, the attracting fixed point. We analyze recurrences in that spirit and remove a gap in previous arguments in the literature, that is, the recursion must be well defined. The latter is known as the Skolem problem. We identify initial values producing a limit equal to the repellent fixed point, show that in every ε -neighborhood of such initial values the recurrence is not well defined, and characterize initial values for which the recurrence is well defined. We give some new examples in that spirit. For example, the correct real result, i.e., the repellent fixed point, may be correctly computed in bfloat, half, single, double, formerly extended precision (80 bit format), binary128 as well as many formats of much higher precision. Rounding errors may be beneficial by introducing some regularizing effect.

Key Words.

recurrences, rounding errors, IEEE-754, different precisions, bfloat, half precision (binary16), single precision (binary32), double precision (binary64), extended precision (binary128), multiple precision, Skolem problem, Pisot sequence

AMS Subject Classifications.

65G50, 11B37

- 370** Perturbation analysis on matrix pencils for two specified eigenpairs of a semisimple eigenvalue with multiplicity two.

Sk. Safique Ahmad and Prince Kanhya.

Abstract.

In this paper, we derive backward error formulas of two approximate eigenpairs of a semisimple eigenvalue with multiplicity two for structured and unstructured matrix pencils. We also construct the minimal structured perturbations with respect to the *Frobenius norm* such that these approximate eigenpairs become exact eigenpairs of an appropriately perturbed matrix pencil. The structures we consider include *T-symmetric/T-skew-symmetric*, *Hermitian/skew-Hermitian*, *T-even/T-odd*, and *H-even/H-odd* matrix pencils. Further, we establish various relationships between the backward error of a single approximate eigenpair and the backward error of two approximate eigenpairs of a semisimple eigenvalue with multiplicity two.

Key Words.

multiple eigenvalue, semisimple eigenvalue, defective eigenvalue, structured generalized eigenvalue problem, eigenpair backward error

AMS Subject Classifications.

65F15, 15A18, 65F35, 15A12

- 391** Finite difference schemes for an axisymmetric nonlinear heat equation with blow-up.

*Chien-Hong Cho and Hisashi Okamoto.***Abstract.**

We study finite difference schemes for axisymmetric blow-up solutions of a nonlinear heat equation in higher spatial dimensions. The phenomenology of blow-up in higher-dimensional space is much more complex than that in one space dimension. To obtain a more complete picture for such phenomena from computational results, it is useful to know the technical details of the numerical schemes for higher spatial dimensions. Since first-order differentiation appears in the differential equation, we pay special attention to it. A sufficient condition for stability is derived. In addition to the convergence of the numerical blow-up time, certain blow-up behaviors, such as blow-up sets and blow-up in the L^p -norm, are taken into consideration. It is sometimes experienced that a certain property of solutions of a partial differential equation may be lost by a faithfully constructed convergent numerical scheme. The phenomenon of one-point blow-up is a typical example in the numerical analysis of blow-up problems. We prove that our scheme can preserve such a property. It is also remarkable that the L^p -norm ($1 \leq p < \infty$) of the solution of the nonlinear heat equation may blow up simultaneously with the L^∞ -norm or remains bounded in $[0, T)$, where T denotes the blow-up time of the L^∞ -norm. We propose a systematic way to compute numerical evidence of the L^p -norm blow-up. The computational results are also analyzed. Moreover, we prove an abstract theorem which shows the relationship between the numerical L^p -norm blow-up and the exact L^p -norm blow-up. Numerical examples for higher-dimensional blow-up solutions are presented and discussed.

Key Words.blow-up, finite difference method, nonlinear heat equation, L^p -norm blow-up**AMS Subject Classifications.**

65M06, 65M12

- 416** Approximation of weakly singular integral equations by sinc projection methods.

*Khadijeh Nedaiasl.***Abstract.**

In this paper, two numerical schemes for a nonlinear integral equation of Fredholm type with weakly singular kernel are studied. These numerical methods blend collocation, convolution, and approximations based on sinc basis functions with iterative schemes like the steepest descent and Newton's method, involving the solution of a nonlinear system of equations. Exponential rate of convergence for the convolution scheme is shown and collocation method is analyzed. Numerical experiments are presented to illustrate the sharpness of the theoretical estimates and the sensitivity of the solutions with respect to some parameters in the equations. The comparison between the schemes indicates that the sinc convolution method is more effective.

Key Words.

Fredholm integral equation, Urysohn integral operator, weak singularity, convolution method, collocation method

AMS Subject Classifications.

45B05, 45E99, 65J15, 65R60.

- 431** Analysis of Krylov subspace approximation to large-scale differential Riccati equations.

*Antti Koskela and Hermann Mena.***Abstract.**

We consider a Krylov subspace approximation method for the symmetric differential Riccati equation $\dot{X} = AX + XA^T + Q - XSX$, $X(0) = X_0$. The method we consider is based on projecting the large-scale equation onto a Krylov subspace spanned by the matrix A and the low-rank factors of X_0 and Q . We prove that the method is structure preserving in the sense that it preserves two important properties of the exact flow, namely the positivity of the exact flow and also the property of monotonicity. We provide a theoretical a priori error analysis that shows superlinear convergence of the method. Moreover, we derive an a posteriori error estimate that is shown to be effective in numerical examples.

Key Words.

differential Riccati equations, LQR optimal control problems, large-scale ordinary differential equations, Krylov subspace methods, matrix exponential, exponential integrators, model order reduction, low-rank approximation

AMS Subject Classifications.

65F10, 65F60, 65L20, 65M22, 93A15, 93C05

- 455** An a-priori error analysis for discontinuous Lagrangian finite elements applied to nonconforming dual-mixed formulations: Poisson and Stokes problems.

*Tomás P. Barrios and Rommel Bustinza.***Abstract.**

In this paper, we discuss the well-posedness of a mixed discontinuous Galerkin (DG) scheme for the Poisson and Stokes problems in 2D, considering only piecewise Lagrangian finite elements. The complication here lies in the fact that the classical Babuška-Brezzi theory is difficult to verify for low-order finite elements, so we proceed in a non-standard way. First, we prove uniqueness, and then we apply a discrete version of Fredholm's alternative theorem to ensure existence. The a-priori error analysis is done by introducing suitable projections of the exact solution. As a result, we prove that the method is convergent, and, under standard additional regularity assumptions on the exact solution, the optimal rate of convergence of the method is guaranteed.

Key Words.

discontinuous Galerkin, Lagrange shape functions, a-priori error estimates

AMS Subject Classifications.

65N30; 65N12; 65N15

- 480** On pole-swapping algorithms for the eigenvalue problem.

*Daan Camps, Thomas Mach, Raf Vandebril, and David S. Watkins.***Abstract.**

Pole-swapping algorithms, which are generalizations of the QZ algorithm for the

generalized eigenvalue problem, are studied. A new modular (and therefore more flexible) convergence theory that applies to all pole-swapping algorithms is developed. A key component of all such algorithms is a procedure that swaps two adjacent eigenvalues in a triangular pencil. An improved swapping routine is developed, and its superiority over existing methods is demonstrated by a backward error analysis and numerical tests. The modularity of the new convergence theory and the generality of the pole-swapping approach shed new light on bi-directional chasing algorithms, optimally packed shifts, and bulge pencils, and allow the design of novel algorithms.

Key Words.

eigenvalue, QZ algorithm, pole swapping, convergence

AMS Subject Classifications.

65F15, 15A18

- 509** Primal-dual block-proximal splitting for a class of non-convex problems.
Stanislav Mazurenko, Jyrki Jauhainen, and Tuomo Valkonen.

Abstract.

We develop block structure-adapted primal-dual algorithms for non-convex non-smooth optimisation problems, whose objectives can be written as compositions $G(x) + F(K(x))$ of non-smooth block-separable convex functions G and F with a nonlinear Lipschitz-differentiable operator K . Our methods are refinements of the nonlinear primal-dual proximal splitting method for such problems without the block structure, which itself is based on the primal-dual proximal splitting method of Chambolle and Pock for convex problems. We propose individual step length parameters and acceleration rules for each of the primal and dual blocks of the problem. This allows them to converge faster by adapting to the structure of the problem. For the squared distance of the iterates to a critical point, we show local $O(1/N)$, $O(1/N^2)$, and linear rates under varying conditions and choices of the step length parameters. Finally, we demonstrate the performance of the methods for the practical inverse problems of diffusion tensor imaging and electrical impedance tomography.

Key Words.

primal-dual algorithms, convex optimization, non-smooth optimization, step length

AMS Subject Classifications.

49M29, 65K10, 90C30

- 553** Analysis of BDDC algorithms for Stokes problems with hybridizable discontinuous Galerkin discretizations.
Xuemin Tu, Bin Wang, and Jinjin Zhang.

Abstract.

The BDDC (balancing domain decomposition by constraints) methods have been applied to solve the saddle point problem arising from a hybridizable discontinuous Galerkin (HDG) discretization of the incompressible Stokes problem. In the BDDC algorithms, the coarse problem is composed by the edge/face constraints across the subdomain interface for each velocity component. As for the standard approaches of the BDDC algorithms for saddle point problems, these constraints ensure that the BDDC preconditioned conjugate gradient (CG) iterations stay in a subspace where

the preconditioned operator is positive definite. However, there are several popular choices of the local stabilization parameters used in the HDG discretizations. Different stabilization parameters change the properties of the resulting discretized operators, and some special observations and tools are needed in the analysis of the condition numbers of the BDDC preconditioned Stokes operators. In this paper, condition number estimates for different choices of stabilization parameters are provided. Numerical experiments confirm the theory.

Key Words.

discontinuous Galerkin, HDG, domain decomposition, BDDC, Stokes problems, Saddle point problems, benign subspace

AMS Subject Classifications.

65F10, 65N30, 65N55

- 571** Addendum to “On recurrences converging to the wrong limit in finite precision and some new examples”.
Siegfried M. Rump.

Abstract.

In a recent paper [Electron. Trans. Numer. Anal, 52 (2020), pp. 358–369], we analyzed Muller’s famous recurrence, where, for particular initial values, the iteration over real numbers converges to a repellent fixed point, whereas finite precision arithmetic produces a different result, the attracting fixed point. We gave necessary and sufficient conditions for such recurrences to produce only nonzero iterates. In the above-mentioned paper, an example was given where only finitely many terms of the recurrence over \mathbb{R} are well defined, but floating-point evaluation indicates convergence to the attracting fixed point. The input data of that example, however, are not representable in binary floating-point, and the question was posed whether such examples exist with binary representable data. This note answers that question in the affirmative.

Key Words.

recurrences, rounding errors, IEEE-754, exactly representable data, bfloat, half precision (binary16), single precision (binary32), double precision (binary64)

AMS Subject Classifications.

65G50, 11B37

- 576** Mathematical and numerical analysis of an acid-mediated cancer invasion model with nonlinear diffusion.
L. Shangerganesh and J. Manimaran.

Abstract.

In this paper, we study the existence of weak solutions of the nonlinear cancer invasion parabolic system with density-dependent diffusion operators. To establish the existence result, we use regularization, the Faedo-Galerkin approximation method, some a priori estimates, and compactness arguments. Furthermore in this paper, we present results of numerical simulations for the considered invasion system with various nonlinear density-dependent diffusion operators. A standard Galerkin finite element method with the backward Euler algorithm in time is used as a numerical tool to discretize the given cancer invasion parabolic system. The theoretical results are validated by numerical examples.

Key Words.

cancer invasion, density-dependent diffusion, Faedo-Galerkin approximation, finite element method

AMS Subject Classifications.

35D30, 65M60, 35K57