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An algorithm for the numerical solution of differential equations of fractional order. *Kai Diethelm*.

Abstract. Differential equations involving derivatives of non-integer order have shown to be adequate models for various physical phenomena in areas like damping laws, diffusion processes, etc. A small number of algorithms for the numerical solution of these equations has been suggested, but mainly without any error estimates. In this paper, we propose an implicit algorithm for the approximate solution of an important class of these equations. The algorithm is based on a quadrature formula approach. Error estimates and numerical examples are given.

Key words. Fractional derivative, Riemann-Liouville derivative, differential equation, numerical solution, quadrature formula, implicit method.

AMS(MOS) subject classification. 26A33, 65L70, 65L05.

Files.

vol.5.1997/pp1-6.dir/pp1-6.ps. vol.5.1997/pp1-6.dir/pp1-6.pdf.

Forward References.

7 On a converse of Laguerre's Theorem. *Thomas Craven and George Csordas*.

Abstract. The problem of characterizing all real sequences $\{\gamma_k\}_{k=0}^{\infty}$ with the property that if $p(x) = \sum_{k=0}^{n} a_k x^k$ is any real polynomial, then $\sum_{k=0}^{n} \gamma_k a_k x^k$ has no more nonreal zeros than p(x), remains open. Recently, the authors solved this problem under the additional assumption that the sequences $\{\gamma_k\}_{k=0}^{\infty}$, with the aforementioned property, can be interpolated by polynomials. The purpose of this paper is to extend this result to certain transcendental entire functions. In particular, the main result establishes a converse of a classical theorem of Laguerre for these transcendental entire functions.

Key words. Laguerre–Pólya class, entire functions, zero distribution, multiplier sequences.

AMS(MOS) subject classification. 26C10, 30D15, 30D10.

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vol.5.1997/pp7-17.dir/pp7-17.ps. vol.5.1997/pp7-17.dir/pp7-17.pdf.

Forward References.

18 Circulant preconditioners for convolution-like integral equations with higher-order quadrature rules. *Michael K. Ng.*

Abstract. In this paper, we consider solving matrix systems arising from the discretization of convolution-like integral equations by preconditioned conjugate gradient (PCG) methods. Circulant integral operators as preconditioners have been proposed and studied. However, the discretization of these circulant preconditioned

equations by employing higher-order quadratures leads to matrix systems that cannot be solved efficiently by using fast Fourier transforms (FFTs). The aim of this paper is to propose "inverted" circulant preconditioners for convolution-like integral equations. The discretization of these preconditioned integral equations by higher-order quadratures leads to matrix systems that involve only Toeplitz, circulant and diagonal matrix-vector multiplications, and hence can be computed efficiently by FFTs in each iteration. Numerical examples are given to illustrate the fast convergence of the method and the improvement of the accuracy of the computed solutions with using higher-order quadratures. We also apply our method to solve the convolution-like equation arising from the linear least squares estimation in signal processing.

Key words. Integral equations, displacement kernel, quadratures, circulant matrices, Toeplitz matrices, fast Fourier transforms, signal processing.

AMS(MOS) subject classification. 45E10, 45L10, 65R20, 65J10.

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vol.5.1997/pp18-28.dir/pp18-28.ps. vol.5.1997/pp18-28.dir/pp18-28.pdf.

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A new algorithm for the SVD of a long product of matrices and the stability of products. *David E. Stewart*.

Abstract. Lyapunov exponents can be estimated by accurately computing the singular values of long products of matrices, with perhaps 1000 or more factor matrices. These products have extremely large ratios between the largest and smallest eigenvalues. A variant of Rutishauser's Cholesky LR algorithm for computing eigenvalues of symmetric matrices is used to obtain a new algorithm for computing the singular values and vectors of long products of matrices with small backward error in the factor matrices. The basic product SVD algorithm can also be accelerated using hyperbolic Givens' rotations. The method is competitive with Jacobi-based methods for certain problems as numerical results indicate.

Key words. SVD, products of matrices, Lyapunov exponents.

AMS(MOS) subject classification. 65F15, 34D08.

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vol.5.1997/pp29-47.dir/pp29-47.ps. vol.5.1997/pp29-47.dir/pp29-47.pdf.

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Asynchronous weighted additive Schwarz methods. Andreas Frommer, Hartmut Schwandt, and Daniel B. Szyld.

Abstract. A class of asynchronous Schwarz methods for the parallel solution of nonsingular linear systems of the form Ax = f is investigated. This class includes, in particular, an asynchronous algebraic Schwarz method as well as asynchronous multisplitting. Theorems are obtained demonstrating convergence for the cases when A^{-1} is nonnegative and when A is an H-matrix. The results shown are for both the situations with or without overlap between the domains in which an underlying mesh is divided, if such a mesh exists. Numerical experiments on systems of up to over ten million variables on up to 256 processors are presented. They illustrate the convergence properties of the method, as well as the fact that when the

domains are not all of the same size, the asynchronous method can be up to 50% faster than the corresponding synchronous one.

Key words. Asynchronous methods, monotone matrices, H-matrices, linear system, parallel algorithms, multisplittings, additive Schwartz.

AMS(MOS) subject classification. 65F10, 65Y05.

Files.

vol.5.1997/pp48-61.dir/pp48-61.ps. vol.5.1997/pp48-61.dir/pp48-61.pdf.

Forward References.

62 Arnoldi-Faber method for large non hermitian eigenvalue problem. *Vincent Heuveline and Miloud Sadkane.*

Abstract. We propose a restarted Arnoldi's method with Faber polynomials and discuss its use for computing the rightmost eigenvalues of large non hermitian matrices. We illustrate, with the help of some practical test problems, the benefit obtained from the Faber acceleration by comparing this method with the Chebyshev based acceleration. A comparison with the implicitly restarted Arnoldi method is also reported.

Key words. Krylov space, block Arnoldi, Faber polynomials, Schwarz-Christoffel.

AMS(MOS) subject classification. 65F10.

Files.

vol.5.1997/pp62-76.dir/pp62-76.ps. vol.5.1997/pp62-76.dir/pp62-76.pdf.

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77 An analysis of the pole placement problem II. *Volker Mehrmann and Hongguo Xu*.

Abstract. For the solution of the multi-input pole placement problem we derive explicit formulas for the subspace from which the feedback gain matrix can be chosen and for the feedback gain as well as the eigenvector matrix of the closed-loop system. We discuss which Jordan structures can be assigned and also when diagonalizability can be achieved. Based on these formulas we study the conditioning of the pole-placement problem in terms of perturbations in the data and show how the conditioning depends on the condition number of the closed loop eigenvector matrix, the norm of the feedback matrix and the distance to uncontrollability.

Key words. pole placement, condition number, perturbation theory,Jordan form, explicit formulas, Cauchy matrix, Vandermonde matrix, stabilization, feedback gain, distance to uncontrollability.

AMS(MOS) subject classification. 65F15, 65F35, 65G05, 93B05, 93B55.

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