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Contents

Abstract.
The development of preconditioners for PDE-constrained optimization problems is a field of numerical analysis which has recently generated much interest. One class of problems which has been investigated in particular is that of Stokes control problems, that is, the problem of minimizing a functional with the Stokes (or Navier-Stokes) equations as constraints. In this manuscript, we present an approach for preconditioning Stokes control problems using preconditioners for the Poisson control problem and, crucially, the application of a commutator argument. This methodology leads to two block diagonal preconditioners for the problem, one of which was previously derived by W. Zulehner in 2011 [SIAM J. Matrix Anal. Appl., 32 (2011), pp. 536–560] using a nonstandard norm argument for this saddle point problem, and the other of which we believe to be new. We also derive two related block triangular preconditioners using the same methodology and present numerical results to demonstrate the performance of the four preconditioners in practice.

Key Words.
PDE-constrained optimization, Stokes control, saddle point system, preconditioning, Schur complement, commutator

AMS Subject Classifications.
65F08, 65F10, 65F50, 76D07, 76D55, 93C20

Abstract.
We show in this paper that the roots x_1 and x_2 of a scalar quadratic polynomial $ax^2 + bx + c = 0$ with real or complex coefficients a, b, c can be computed in an element-wise mixed stable manner, measured in a relative sense. We also show that this is a stronger property than norm-wise backward stability but weaker than element-wise backward stability. We finally show that there does not exist any method that can compute the roots in an element-wise backward stable sense, which is also illustrated by some numerical experiments.

Key Words.
quadratic polynomial, roots, numerical stability

AMS Subject Classifications.
65G30, 65G50, 65H04

Abstract.
In the framework of large-scale linear discrete ill-posed problems, Krylov projection methods represent an essential tool since their development, which dates back to the early 1950's. In recent years, the use of these methods in a hybrid fashion or to solve Tikhonov regularized problems has received great attention especially for problems involving the restoration of digital images. In this paper we review the fundamental Krylov-Tikhonov techniques based on Lanczos bidiagonalization and the Arnoldi algorithms. Moreover, we study the use of the unsymmetric Lanczos process that, to the best of our knowledge, has just marginally been considered in this setting. Many numerical experiments and comparisons of different methods are presented.

Key Words.
discrete ill-posed problems, Krylov projection methods, Tikhonov regularization, Lanczos bidiagonalization, nonsymmetric Lanczos process, Arnoldi algorithm, discrepancy principle, generalized cross validation, L-curve criterion, Regińska criterion, image deblurring

AMS Subject Classifications.
65F10, 65F22, 65R32.

124 Iterative methods for symmetric outer product tensor decomposition.
Na Li, Carmeliza Navasca, and Christina Glenn.

Abstract.
We study the symmetric outer product for tensors. Specifically, we look at decompositions of a fully (partially) symmetric tensor into a sum of rank-one fully (partially) symmetric tensors. We present an iterative technique for third-order partially symmetric tensors and fourth-order fully and partially symmetric tensors. We include several numerical examples which indicate faster convergence for the new algorithms than for the standard method of alternating least squares.

Key Words.
multilinear algebra, tensor products, factorization of matrices

AMS Subject Classifications.
15A69, 15A23

140 Explicit formulas for Hermite-type interpolation on the circle and applications.
Elías Berriochoa, Alicia Cachafeiro, Jaime Díaz, and Jesús Illán.

Abstract.
In this paper we study two ways of obtaining Laurent polynomials of Hermite interpolation on the unit circle. The corresponding nodal system is constituted by the nth roots of a complex number with modulus one. One of the interpolation formulas is given in terms of an appropriate basis which yields coefficients computable by means of the fast Fourier transform (FFT). The other formula is of barycentric type. As a consequence, we illustrate some applications to the Hermite interpolation problem on $[-1, 1]$. Some numerical tests are presented to emphasize the numerical stability of these formulas.

Key Words.
Hermite interpolation, Laurent polynomials, barycentric formulas, unit circle, Chebyshev polynomials

AMS Subject Classifications.
65D05, 41A05, 33C45

153 Quasi-optimal convergence rates for adaptive boundary element methods with data approximation. Part II: hyper-singular integral equation.
Michael Feischl, Thomas Führer, Michael Karkulik, J. Markus Melenk, and Dirk Praetorius.

Abstract.
We analyze an adaptive boundary element method with fixed-order piecewise polynomials for the hyper-singular integral equation of the Laplace-Neumann problem

in 2D and 3D which incorporates the approximation of the given Neumann data into the overall adaptive scheme. The adaptivity is driven by some residual-error estimator plus data oscillation terms. We prove convergence with quasi-optimal rates. Numerical experiments underline the theoretical results.

Key Words.
boundary element method, hyper-singular integral equation, a posteriori error estimate, adaptive algorithm, convergence, optimality

AMS Subject Classifications.
65N30, 65N15, 65N38

177 Randomized methods for rank-deficient linear systems.
Josef Sifuentes, Zydrunas Gimbutas, and Leslie Greengard.

Abstract.
We present a simple, accurate method for solving consistent, rank-deficient linear systems, with or without additional rank-completing constraints. Such problems arise in a variety of applications such as the computation of the eigenvectors of a matrix corresponding to a known eigenvalue. The method is based on elementary linear algebra combined with the observation that if the matrix is rank-k deficient, then a random rank-k perturbation yields a nonsingular matrix with probability close to 1.

Key Words.
rank-deficient systems, null space, null vectors, eigenvectors, randomized algorithms, integral equations

AMS Subject Classifications.
15A03, 15A12, 15A18, 65F15, 65F99

189 Fast solution of boundary integral equations with the generalized Neumann kernel.
Mohamed M. S. Nasser.

Abstract.
A fast method for solving boundary integral equations with the generalized Neumann kernel and the adjoint generalized Neumann kernel is presented. The complexity of the developed method is $O((m+1)n \ln n)$ for the integral equation with the generalized Neumann kernel and $O((m+1)n)$ for the integral equation with the adjoint generalized Neumann kernel, where $m+1$ is the multiplicity of the multiply connected domain and n is the number of nodes in the discretization of each boundary component. Numerical results illustrate that the method gives accurate results even for domains of very high connectivity, domains with piecewise smooth boundaries, domains with close-to-touching boundaries, and domains of real world problems.

Key Words.
generalized Neumann kernel, boundary integral equations, Nyström method, Fast Multipole Method, GMRES, numerical conformal mapping

AMS Subject Classifications.
45B05, 65R20, 30C30

230 BEM-based finite element tearing and interconnecting methods.
Clemens Hofreither, Ulrich Langer, and Clemens Pechstein.

Abstract.
We present efficient domain decomposition solvers for a class of non-standard finite element methods (FEM). These methods utilize PDE-harmonic trial functions in every element of a polyhedral mesh and use boundary element techniques locally in order to assemble the finite element stiffness matrices. For these reasons, the terms BEM-based FEM or Trefftz-FEM are sometimes used in connection with this method. In the present paper, we show that finite element tearing and interconnecting (FETI) methods can be used to solve the resulting linear systems in a quasi-optimal, robust, and parallel manner. Spectral equivalences between certain approximations of element-local Steklov-Poincaré operators play a central role in transferring the known convergence results for FETI to this new method. The theoretical results are supplemented by numerical tests confirming the theoretical predictions.

Key Words.
finite elements, boundary elements, BEM-based FEM, domain decomposition, FETI, BETI, Trefftz methods, polyhedral meshes

AMS Subject Classifications.
65F10, 65N22, 65N30, 65N38

250 A convergent linear finite element scheme for the Maxwell-Landau-Lifshitz-Gilbert equations.
Lubomir Baňas, Marcus Page, and Dirk Praetorius.

Abstract.
We consider the lowest-order finite element discretization of the nonlinear system of Maxwell's and Landau-Lifshitz-Gilbert equations (MLLG). Two algorithms are proposed to numerically solve this problem, both of which only require the solution of at most two linear systems per time step. One of the algorithms is decoupled in the sense that it consists of the sequential computation of the magnetization and *afterwards* the magnetic and electric field. Under some mild assumptions on the effective field, we show that both algorithms converge towards weak solutions of the MLLG system. Numerical experiments for a micromagnetic benchmark problem demonstrate the performance of the proposed algorithms.

Key Words.
Maxwell-LLG, linear scheme, ferromagnetism, convergence

AMS Subject Classifications.
65N30, 65N50

271 On the discrete extension of Markov's theorem on monotonicity of zeros.
Kenier Castillo and Fernando R. Rafaeli.

Abstract.
Motivated by an open problem proposed by M. E. H. Ismail in his monograph "Classical and quantum orthogonal polynomials in one variable" (Cambridge University Press, 2005), we study the behavior of zeros of orthogonal polynomials associated with a positive measure on $[a, b] \subseteq \mathbb{R}$ which is modified by adding a mass

at $c \in \mathbb{R} \setminus (a,b)$. We prove that the zeros of the corresponding polynomials are strictly increasing functions of c. Moreover, we establish their asymptotics when c tends to infinity or minus infinity, and it is shown that the rate of convergence is of order $1/c$.

Key Words.
orthogonal polynomials on the real line, Uvarov's transformation, Markov's theorem, monotonicity of zeros, asymptotic behavior, speed of convergence

AMS Subject Classifications.
33C45, 30C15

281 Fast algorithms for spectral differentiation matrices.
Jared L. Aurentz.

Abstract.
Recently Olver and Townsend presented a fast spectral method that relies on bases of ultraspherical polynomials to give differentiation matrices that are almost banded. The almost-banded structure allowed them to develop efficient algorithms for solving certain discretized systems in linear time. We show that one can also design fast algorithms for standard spectral methods because the underlying matrices, though dense, have the same rank structure as those of Olver and Townsend.

Key Words.
spectral methods, rank-structured matrices

AMS Subject Classifications.
65N35, 33C45, 65F05

289 An implicit finite difference approximation for the solution of the diffusion equation with distributed order in time.
N. J. Ford, M. L. Morgado, and M. Rebelo.

Abstract.
In this paper we are concerned with the numerical solution of a diffusion equation in which the time derivative is of non-integer order, i.e., in the interval $(0,1)$. An implicit numerical method is presented and its unconditional stability and convergence are proved. Two numerical examples are provided to illustrate the obtained theoretical results.

Key Words.
Caputo derivative, fractional differential equation, subdiffusion, finite difference method, distributed order differential equation

AMS Subject Classifications.
35R11, 65M06, 65M12

306 Roundoff error analysis of the CholeskyQR2 algorithm.
Yusaku Yamamoto, Yuji Nakatsukasa, Yuka Yanagisawa, and Takeshi Fukaya.

Abstract.
We consider the QR decomposition of an $m \times n$ matrix X with full column rank, where $m \geq n$. Among the many algorithms available, the Cholesky QR algorithm is ideal from the viewpoint of high performance computing since it consists entirely

of standard level 3 BLAS operations with large matrix sizes, and requires only one reduce and broadcast in parallel environments. Unfortunately, it is well-known that the algorithm is not numerically stable and the deviation from orthogonality of the computed Q factor is of order $O((\kappa_2(X))^2\mathbf{u})$, where $\kappa_2(X)$ is the 2-norm condition number of X and $\mathbf{u}$ is the unit roundoff. In this paper, we show that if the condition number of X is not too large, we can greatly improve the stability by iterating the Cholesky QR algorithm twice. More specifically, if $\kappa_2(X)$ is at most $O(\mathbf{u}^{-\frac{1}{2}})$, both the residual and deviation from orthogonality are shown to be of order $O(\mathbf{u})$. Numerical results support our theoretical analysis.

Key Words.
QR decomposition, Cholesky QR, communication-avoiding algorithms, roundoff error analysis.

AMS Subject Classifications.
15A23, 65F25, 65G50.

327 Fast and stable unitary QR algorithm.
Jared L. Aurentz, Thomas Mach, Raf Vandebril, and David S. Watkins.

Abstract.
A fast Fortran implementation of a variant of Gragg's unitary Hessenberg QR algorithm is presented. It is proved, moreover, that all QR- and QZ-like algorithms for the unitary eigenvalue problems are equivalent. The algorithm is backward stable. Numerical experiments are presented that confirm the backward stability and compare the speed and accuracy of this algorithm with other methods.

Key Words.
eigenvalue, unitary matrix, Francis's QR algorithm, core transformations rotators

AMS Subject Classifications.
65F15, 65H17, 15A18, 15B10

342 The fast bisection eigenvalue method for Hermitian order one quasiseparable matrices and computations of norms.
Yuli Eidelman and Iulian Haimovici.

Abstract.
Since we can evaluate the characteristic polynomial of an $N \times N$ order one quasiseparable Hermitian matrix A in less than $10N$ arithmetical operations by sharpening results and techniques from Eidelman, Gohberg, and Olshevsky [Linear Algebra Appl., 405 (2005), pp. 1–40], we use the Sturm property with bisection to compute all or selected eigenvalues of A. Moreover, linear complexity algorithms are established for computing norms, in particular, the Frobenius norm $\|A\|_F$ and $\|A\|_\infty, \|A\|_1$, and other bounds for the initial interval to be bisected. Upper and lower bounds for eigenvalues are given by the Gershgorin Circle Theorem, and we describe an algorithm with linear complexity to compute them for quasiseparable matrices.

Key Words.
quasiseparable, Hermitian, Sturm property, matrix norm, eigenvalues, bisection

AMS Subject Classifications.
15A18, 15A15, 65F35, 15A57, 65F15

367 Efficient preconditioners for PDE-constrained optimization problem with a multilevel sequentially semiseparable matrix structure.
Yue Qiu, Martin B. van Gijzen, Jan-Willem van Wingerden, Michel Verhaegen, and Cornelis Vuik.

Abstract.
PDE-constrained optimization problems yield a linear saddle-point system that has to be solved. We propose a preconditioner that makes use of the global MSSS structure and a preconditioner that exploits the block MSSS structure of the saddle-point system. For the computation of preconditioners based on MSSS matrix computations, model order reduction algorithms are essential to obtain a low computational complexity. We study two different model order reduction approaches, one is the new approximate balanced truncation with low-rank approximated Gramians for SSS matrices and the other is the standard Hankel blocks approximation algorithm. We test our preconditioners on the problems of optimal control of the convection-diffusion equation in 2D and of the Poisson equation in 3D. For 2D problems, numerical experiments illustrate that both preconditioners have linear computational complexity and the global MSSS preconditioner reduces the number of iterations significantly and needs less computation time. Moreover, the approximate balanced truncation algorithm is computationally cheaper than the Hankel blocks approximation algorithm. Besides the mesh size independent convergence, the global MSSS preconditioner also gives the regularization parameter independent convergence, while the block MSSS preconditioner just gives mesh size independent convergence. For 3D problems, both the block MSSS preconditioner and global MSSS preconditioner give virtually mesh size independent convergence. Furthermore, the global MSSS preonconditioner reduces the number of iterations dramatically compared with the block MSSS preconditioner.

Key Words.
PDE-constrained optimization, saddle-point problem, preconditioners, multilevel sequentially semiseparable matrix, model order reduction, low-rank approximation

AMS Subject Classifications.
15B99, 65Fxx, 93C20, 65Y20

401 An overview of multilevel methods with aggressive coarsening and massive polynomial smoothing.
Jan Brousek, Pavla Franková, Milan Hanuš, Hana Kopincová, Roman Kužel, Radek Tezaur, Petr Vaněk, and Zbyněk Vastl.

Abstract.
We review our two-level and multilevel methods with aggressive coarsening and polynomial smoothing. These methods can be seen as a less expensive and more flexible (in the multilevel case) alternative to domain decomposition methods. The polynomial smoothers employed by the reviewed methods consist of a sequence of Richardson iterations and can be performed using up to n processors, where n is the size of the considered matrix, thereby allowing for a higher level of parallelism than domain decomposition methods.

Key Words.
multigrid, aggressive coarsening, optimal convergence result

AMS Subject Classifications.
65F10,65M55

443 Edge-based Schwarz methods for the Crouzeix-Raviart finite volume element discretization of elliptic problems.
Atle Loneland, Leszek Marcinkowski, and Talal Rahman.

Abstract.
In this paper, we present two variants of the additive Schwarz method for a Crouzeix-Raviart finite volume element (CRFVE) discretization of second-order elliptic problems with discontinuous coefficients, where the discontinuities may be across subdomain boundaries. The preconditioner in one variant is symmetric, while in the other variant it is nonsymmetric. The proposed methods are quasi optimal, in the sense that the convergence of the preconditioned GMRES iteration in both cases depend only poly-logarithmically on the ratio of the subdomain size to the mesh size.

Key Words.
domain decomposition, Crouzeix-Raviart element, additive Schwarz method, finite volume element, GMRES

AMS Subject Classifications.
65F10, 65N22, 65N30, 63N55

462 High-order modified tau method for non-smooth solutions of Abel integral equations.
Payam Mokhtary.

Abstract.
In this paper, the spectral Tau method and generalized Jacobi functions are fruitfully combined to approximate Abel integral equations with solutions that may have singularities (non-smooth solutions) at the origin. In an earlier work of P. Mokhtary and F. Ghoreishi [Electron. Trans. Numer. Anal., 41 (2014), pp. 289–305], a regularization process was used to handle the high-order Tau method based on classical Jacobi polynomials for the numerical solution of Abel integral equations. However, it was found that this scheme makes the resulting equation and its Tau approximation more complicated. In this work, we introduce and analyze a new modified Tau method for the numerical solution of Abel integral equations with non-smooth solutions. The main advantage of this method is that it gains a high order of accuracy without adopting any regularization process. Illustrative examples are included to demonstrate the validity and applicability of the proposed technique.

Key Words.
modified Tau method, generalized Jacobi functions, Abel integral equations.

AMS Subject Classifications.
45E10, 41A25.

472 Algebraic distance for anisotropic diffusion problems: multilevel results.
Achi Brandt, James Brannick, Karsten Kahl, and Irene Livshits.

Abstract.
In this paper, we motivate, discuss the implementation, and present the resulting numerics for a new definition of strength of connection which uses the notion of *algebraic distance* as defined originally in the bootstrap algebraic multigrid framework (BAMG). We use this algebraic distance measure together with compatible relaxation and least-squares interpolation to derive an algorithm for choosing suitable

coarse grids and accurate interpolation operators for algebraic multigrid algorithms. The main tool of the proposed strength measure is the least-squares functional defined by using a set of test vectors that in general is computed using the bootstrap process. The motivating application is the anisotropic diffusion problem, in particular, with non-grid aligned anisotropy. We demonstrate numerically that the measure yields a robust technique for determining strength of connectivity among variables for both two-grid and multigrid bootstrap algebraic multigrid methods. The proposed algebraic distance measure can also be used in any other coarsening procedure assuming that a rich enough set of near-kernel components of the matrix for the targeted system is known or is computed as in the bootstrap process.

Key Words.
bootstrap algebraic multigrid, least-squares interpolation, algebraic distances, strength of connection

AMS Subject Classifications.
65N55, 65N22 , 65F10

Abstract.
A bound is obtained for the condition number of a two-level overlapping Schwarz algorithm for problems posed in $H(\mathrm{curl})$ in two dimensions, where the subdomains are only assumed to be John subdomains. The coarse space is based on energy minimization and its dimension equals the number of interior subdomain edges. Local direct solvers are used on the overlapping subdomains. Our bound depends only on a few geometric parameters of the decomposition. This bound is independent of jumps in the coefficients across the interface between the subdomains for most of the different cases considered. Numerical experiments that verify the result are shown, including some with subdomains with fractal edges and others obtained by a mesh partitioner.

Key Words.
Domain decomposition, overlapping Schwarz algorithms, preconditioners, irregular subdomain boundaries, H(curl), Maxwell's equations, discontinuous coefficients

AMS Subject Classifications.
65N55, 65N30, 65F10, 35Q60

Abstract.
A recycling Krylov subspace method for the solution of a sequence of self-adjoint linear systems is proposed. Such problems appear, for example, in the Newton process for solving nonlinear equations. Ritz vectors are automatically extracted from one MINRES run and then used for self-adjoint deflation in the next. The method is designed to work with arbitrary inner products and arbitrary self-adjoint positive-definite preconditioners whose inverse can be computed with high accuracy. Numerical experiments with nonlinear Schrödinger equations indicate a substantial decrease in computation time when recycling is used.

Key Words.
Krylov subspace methods, MINRES, deflation, Ritz vector recycling, nonlinear Schrödinger equations, Ginzburg–Landau equations

AMS Subject Classifications.
65F10, 65F08, 35Q55, 35Q56

Abstract.
This article describes possible causes of natural laminar–turbulent transition in circular pipe flow. Our starting points are the observations that under natural disturbance conditions, transition appears to take place only in the developing entrance region, as observed in Reynolds' color-band experiments, and that the critical Reynolds number R_c has a minimum value of about 2000 when using a sharp-edged uniform radius pipe, as observed in our earlier color-band experiments. The entrance region is defined as the region from the pipe inlet to the point where the inlet flow fully develops into Hagen-Poiseuille flow for a sharp-edged entrance pipe. In the case of a bell-mouth entrance pipe, the entrance region includes the bell-mouth entrance region. We derive for the entrance region a new ratio of the increase in kinetic energy flux ($\varDelta$KE flux) to a wall effect, where the wall effect is the radial wall power (R-Wall-Power) exerted on the wall by the radial component of the viscous term in the Navier-Stokes equations. In dimensionless form, $\varDelta$KE flux is a constant, although R-Wall-Power decreases as the Reynolds number Re increases. Our previous calculations for the case of a sharp-edged entrance pipe indicate that $\varDelta$KE flux $\approx$ total R-Wall-Power (T-R-Wall-Power) at Re $\approx$ 2000. Accordingly, our hypothesis is that R_c can be derived from the relation between $\varDelta$KE flux and T-R-Wall-Power. We discuss, moreover, whether or not this criterion can apply to different entrance geometries such as the bell-mouth entrances considered by Reynolds.

Key Words.
hydrodynamic stability, mesh refinement, thermodynamics

AMS Subject Classifications.
76E05, 65M50, 80A05

Abstract.
We show in this paper how to properly discretize optimized Schwarz methods for the time-harmonic Maxwell's equations in two and three spatial dimensions using a discontinuous Galerkin (DG) method. Due to the multiple traces between elements in the DG formulation, it is not clear a priori how the more sophisticated transmission conditions in optimized Schwarz methods should be discretized, and the most natural approach, at convergence of the Schwarz method, does not lead to the monodomain DG solution, which implies that for such discretizations, the DG error estimates do not hold when the Schwarz method has converged. We present here a consistent discretization of the transmission conditions in the framework of a DG

weak formulation, for which we prove that the multidomain and monodomain solutions for the Maxwell's equations are the same. We illustrate our results with several numerical experiments of propagation problems in homogeneous and heterogeneous media.

Key Words.
computational electromagnetism, time-harmonic Maxwell's equations, Discontinuous Galerkin method, optimized Schwarz methods, transmission conditions.

AMS Subject Classifications.
65M55, 65F10, 65N22

593 SVD of Hankel matrices in Vandermonde-Cauchy product form.
Zlatko Drmač.

Abstract.
Structured matrices of Cauchy, Vandermonde, Hankel, Toeplitz, and other types arise in a variety of applications, and their SVD decomposition provides key information, e.g., in various rational approximation tasks. In particular, Hankel matrices play an important role in the Adamyan-Arov-Krein and Carathéodory-Feyér rational approximation theories as well as in various applications in signal processing and control theory. This paper proposes new algorithms to compute the SVD of a Hankel matrix given implicitly as the product $\mathcal{V}^T D\mathcal{V}$, where $\mathcal{V}$ is a complex Vandermonde matrix and D is a diagonal matrix. The key steps are the discrete Fourier transform and the computation of the SVD of $\mathcal{C}^T \widetilde{D}\mathcal{C}$, where $\mathcal{C}$ is a Cauchy matrix and $\widetilde{D}$ is diagonal. This SVD is computed by a specially tailored version of the Jacobi SVD for products of matrices. Error and perturbation analysis and numerical experiments confirm the robustness of the proposed algorithms, capable of computing to high relative accuracy all singular values in the full range of machine numbers.

Key Words.
Cauchy matrix, discrete Fourier transform, eigenvalues, Hankel matrix, Jacobi method, rational approximations, singular value decomposition, Toeplitz matrix, Vandermonde matrix

AMS Subject Classifications.
15A09, 15A12, 15A18, 15A23, 65F15, 65F22, 65F35

624 Perturbation of partitioned linear response eigenvalue problems.
Zhongming Teng, Linzhang Lu, and Ren-Cang Li.

Abstract.
This paper is concerned with bounds for the linear response eigenvalue problem for $H = \begin{bmatrix} 0 & K \\ M & 0 \end{bmatrix}$, where K and M admit a 2×2 block partitioning. Bounds on how the changes of its eigenvalues are obtained when K and M are perturbed. They are of linear order with respect to the diagonal block perturbations and of quadratic order with respect to the off-diagonal block perturbations in K and M. The result is helpful in understanding how the Ritz values move towards eigenvalues in some efficient numerical algorithms for the linear response eigenvalue problem. Numerical experiments are presented to support the analysis.

Key Words.
linear response eigenvalue problem, random phase approximation, perturbation, quadratic perturbation bound

AMS Subject Classifications.
15A42, 65F15

639 Monotone-comonotone approximation by fractal cubic splines and polynomials.
Puthan Veedu Viswanathan and Arya Kumar Bedabrata Chand.

Abstract.
We develop cubic fractal interpolation functions H^α as continuously differentiable α-fractal functions corresponding to the traditional piecewise cubic interpolant H. The elements of the iterated function system are identified so that the class of α-fractal functions f^α reflects the monotonicity and $\mathcal{C}^1$-continuity of the source function f. We use this monotonicity preserving fractal perturbation to: (i) prove the existence of piecewise defined fractal polynomials that are comonotone with a continuous function, (ii) obtain some estimates for monotone and comonotone approximation by fractal polynomials. Drawing on the Fritsch-Carlson theory of monotone cubic interpolation and the developed monotonicity preserving fractal perturbation, we describe an algorithm that constructs a class of monotone cubic fractal interpolation functions H^α for a prescribed set of monotone data. This new class of monotone interpolants provides a large flexibility in the choice of a differentiable monotone interpolant. Furthermore, the proposed class outperforms its traditional non-recursive counterpart in approximation of monotone functions whose first derivatives have varying irregularity/fractality (smooth to nowhere differentiable).

Key Words.
Fractal function, cubic Hermite fractal interpolation function, fractal polynomial, Fritsch-Carlson algorithm, comonotonicity

AMS Subject Classifications.
65D05, 41A29, 41A30, 28A80

660 Polynomial interpolation in nondivision algebras.
Gerhard Opfer.

Abstract.
Algorithms for two types of interpolation polynomials in nondivision algebras are presented. One is based on the Vandermonde matrix, and the other is close to the Newton interpolation scheme. Examples are taken from $\mathbb{R}^4$-algebras. In the Vandermonde case, necessary and sufficient conditions for the existence of interpolation polynomials are given for commutative algebras. For noncommutative algebras, a conjecture is proposed. This conjecture is true for equidistant nodes. It is shown that the Newton form of the interpolation polynomial exists if and only if all node differences are invertible. Several numerical examples are presented.

Key Words.
interpolation polynomials in nondivision algebras, Vandermonde-type polynomials in nondivision algebras, Newton-type polynomials in nondivision algebras, numerical examples of interpolation polynomials in nondivision algebras of $\mathbb{R}^4$

AMS Subject Classifications.
15A66, 1604, 41A05, 65D05